

*Structural systematic  
in the  
phase-diagrams  
of the  
elements*

A simple guide to the world of high pressure  
AIRAPT lecture 2023

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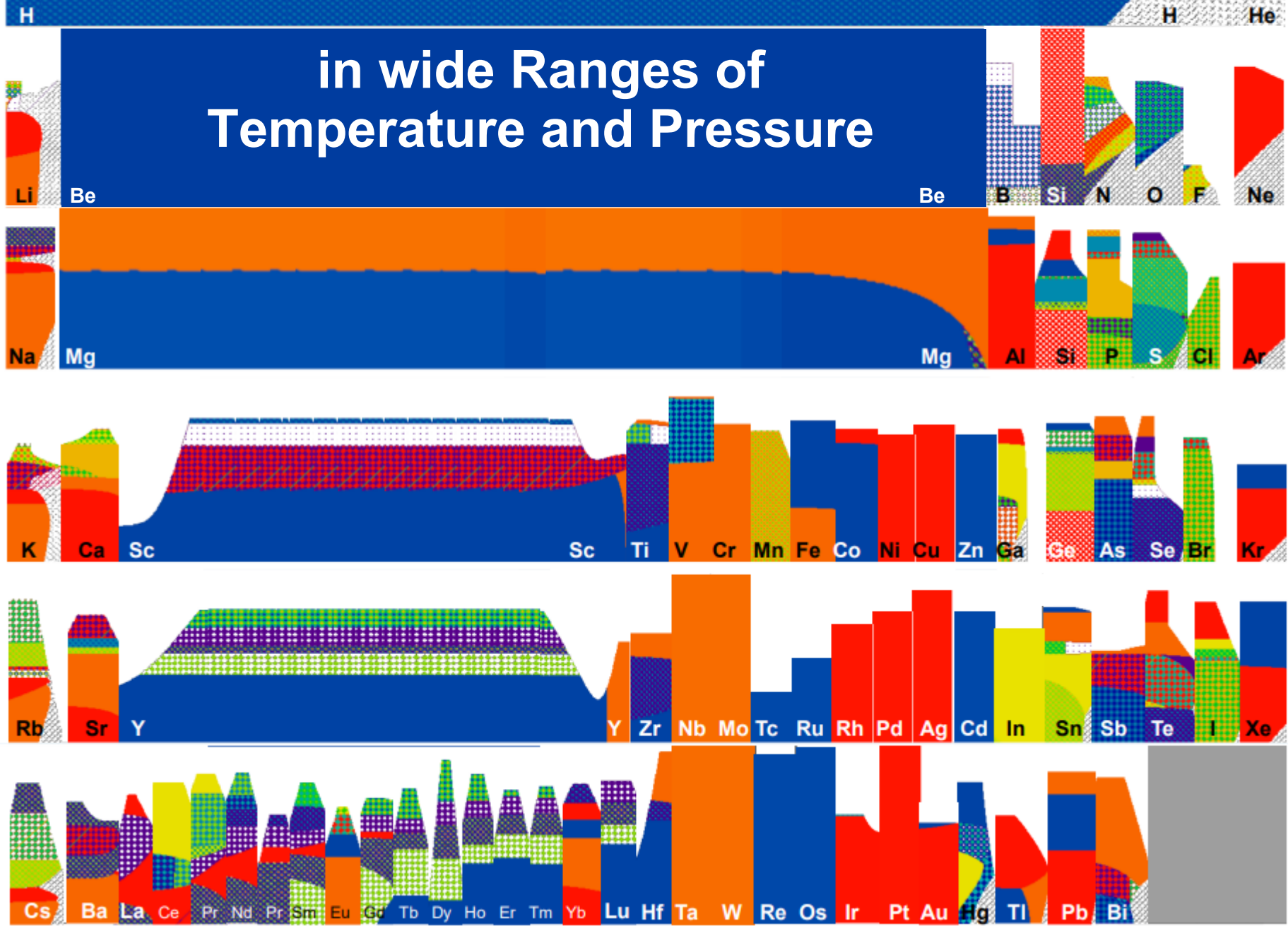
# Crystal Structures of the Elements at ambient Pressure

<b>H</b>																<b>H</b>	<b>He</b>
<b>Li</b>	<b>Be</b>										<b>B</b>	<b>C</b>	<b>N</b>	<b>O</b>	<b>F</b>	<b>Ne</b>	
<b>Na</b>	<b>Mg</b>										<b>Al</b>	<b>Si</b>	<b>P</b>	<b>S</b>	<b>Cl</b>	<b>Ar</b>	
<b>K</b>	<b>Ca</b>	<b>Sc</b>	<b>Ti</b>	<b>V</b>	<b>Cr</b>	<b>Mn</b>	<b>Fe</b>	<b>Co</b>	<b>Ni</b>	<b>Cu</b>	<b>Zn</b>	<b>Ga</b>	<b>Ge</b>	<b>As</b>	<b>Se</b>	<b>Br</b>	<b>Kr</b>
<b>Rb</b>	<b>Sr</b>	<b>Y</b>	<b>Zr</b>	<b>Nb</b>	<b>Mo</b>	<b>Tc</b>	<b>Ru</b>	<b>Rh</b>	<b>Pd</b>	<b>Ag</b>	<b>Cd</b>	<b>In</b>	<b>Sn</b>	<b>Sb</b>	<b>Te</b>	<b>J</b>	<b>Xe</b>
<b>Cs</b>	<b>Ba</b>	<b>La....Lu</b>	<b>Hf</b>	<b>Ta</b>	<b>W</b>	<b>Re</b>	<b>Os</b>	<b>Ir</b>	<b>Pt</b>	<b>Au</b>	<b>Hg</b>	<b>Tl</b>	<b>Pb</b>	<b>Bi</b>			

## Key for the Crystal Structures

cl2 	hP2 	hR105 	cF4 	cF8 	oC8 
cl58 	hR2 	hR3 	tl2 	tl4 	oF128 

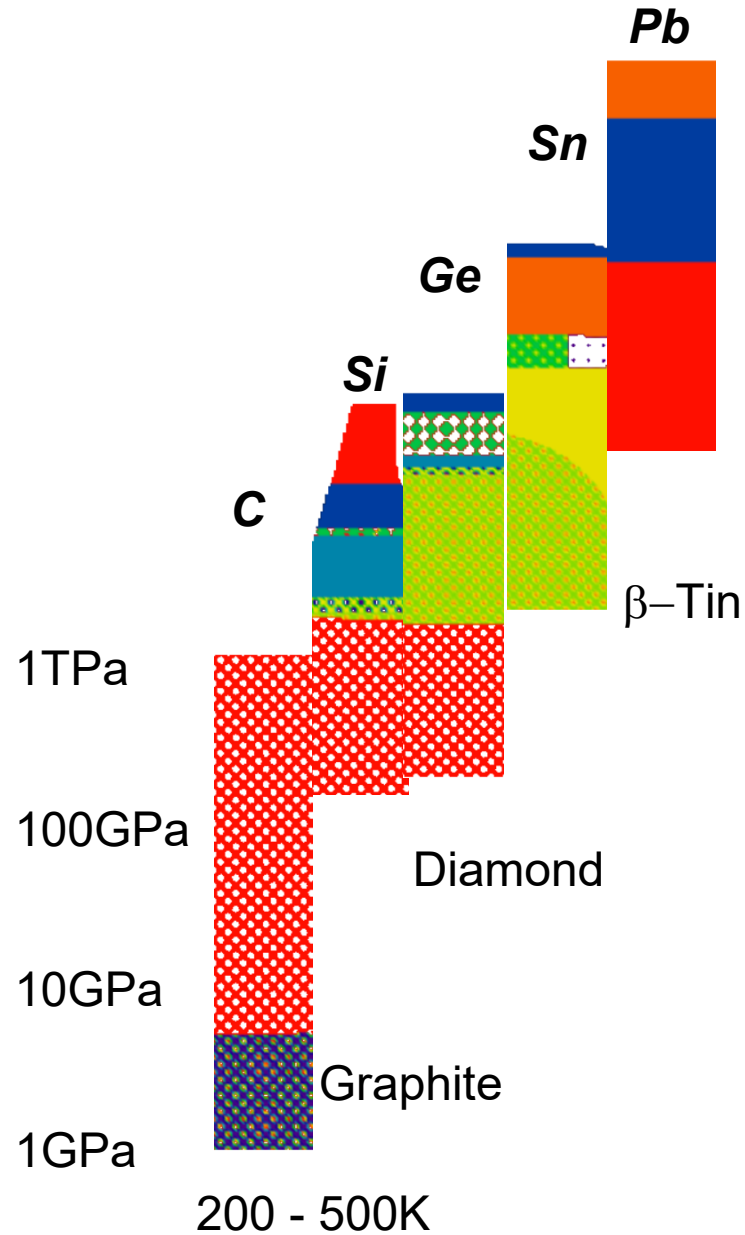
# The colorful World of the Elements



in wide Ranges of  
Temperature and Pressure

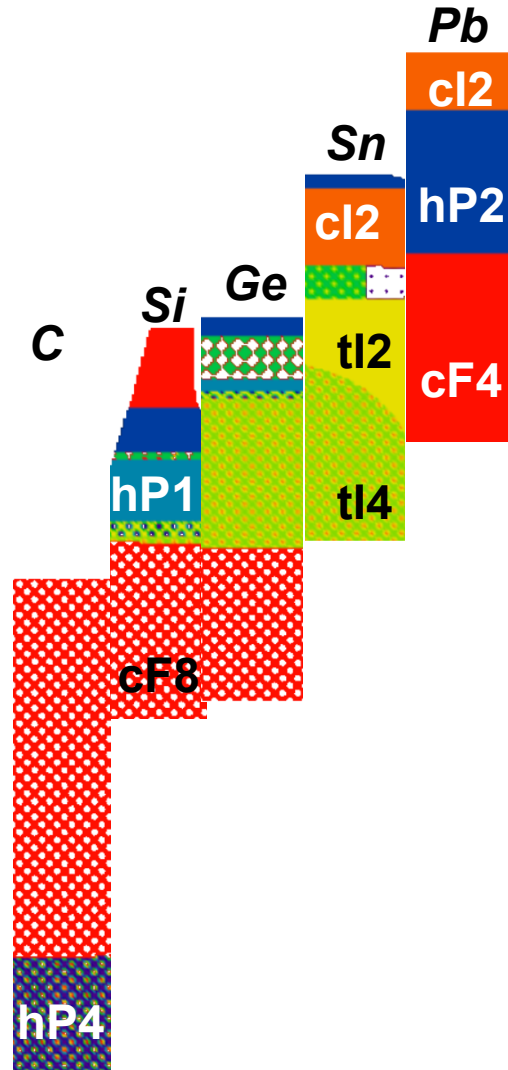
# The Carbon Group

Smaller values for the ratio of atomic versus ionic radii result in high pressure structures.  
Heavier Elements start with smaller radius ratios.



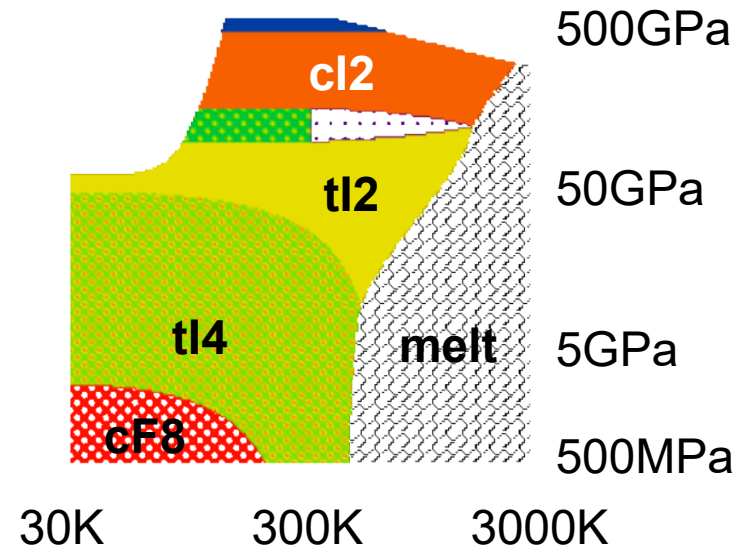
# The Carbon Group

Section 1TPa  
to  
Diagrams 1GPa  
200 to 500K



<b>C</b>	hP4	→	cF8	→	275GPa										
ptr/GPa	3														
<b>Si</b>	cF8	→	tI4	→	oI4	→	hP1	→	oC16	→	hP2	→	cF4	→	250GPa
ptr/GPa	12	13	16	38	42	80									
<b>Ge</b>	cF8	→	tI4	→	oI4	→	hP1	→	oC16	→	hP2	→	>180GPa		
ptr/GPa	11	75	85	102	160										
<b>Sn</b>	cF8	→	tI4	→	tI2	→	oI2	→	cI2	→	hP2	→	>194GPa		
ptr/GPa	0	9.2	32	70	157										
<b>Pb</b>	cF4	→	hP2	→	cI2	→	>272GPa								
ptr/GPa	13	127													

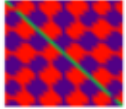
**Sn: 30-3000K, 0.5-500GPa**



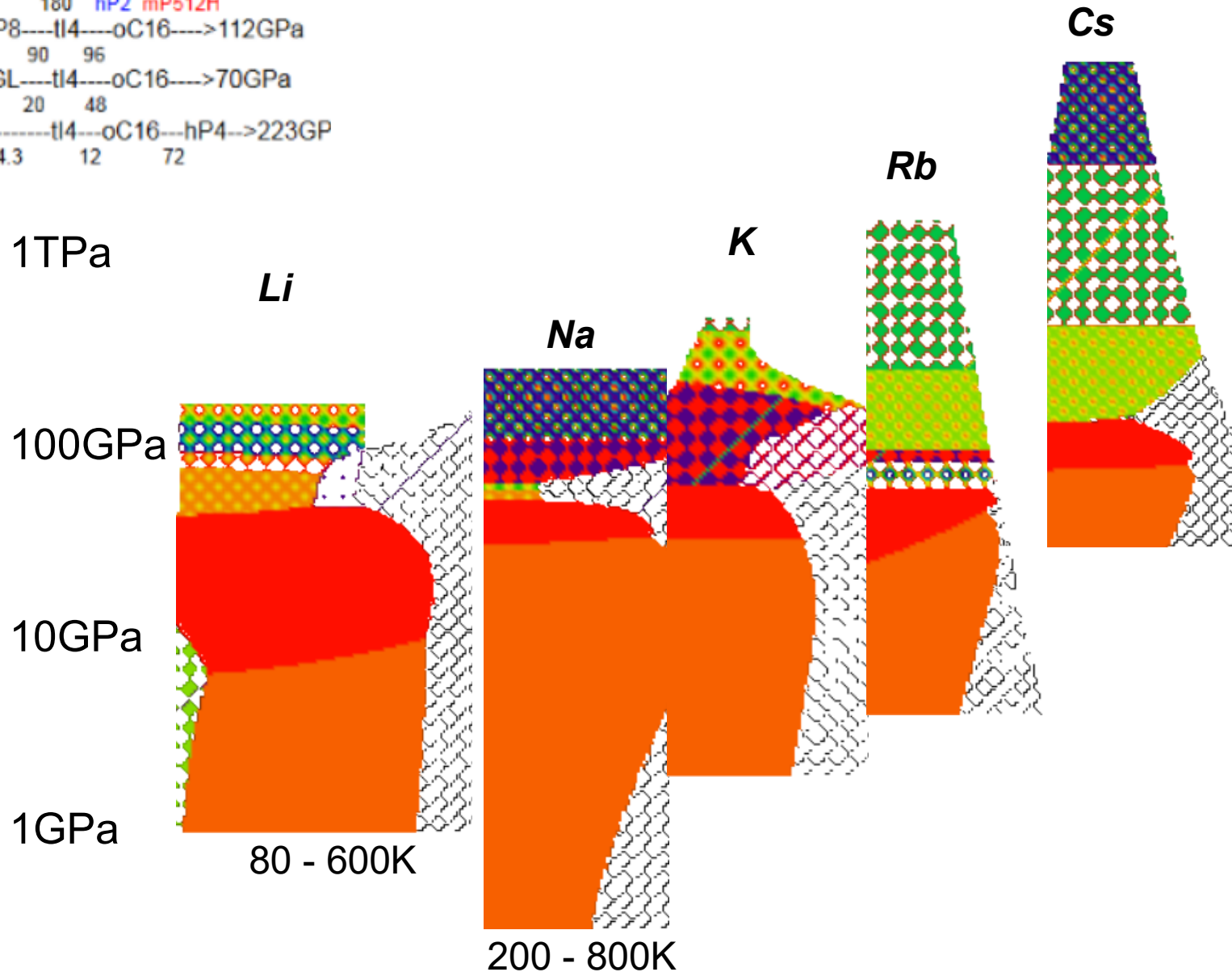
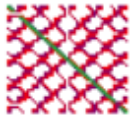
# The Alkaline Metals

<b>Li</b>	cl2	cF4	liquid	oC40	oC24	>116GPa
ptr/GPa	7.5	39		75	90	
200K:	hR1	cl16	oC88	oC40	oC24	
<b>Na</b>	cl2	cF4	cl16	oP8***	HGL	hP4 >215GPa
ptr/GPa	65	104	115	125	180	hP2 mP512H
<b>K</b>	cl2	cF4	HGL	oP8	tl4	oC16 >112GPa
ptr/GPa	11.6	20		54	90	96
<b>Rb</b>	cl2	cF4	oC52	HGL	tl4	oC16 >70GPa
ptr/GPa	7	13	17	20	48	
<b>Cs</b>	cl2	cF4	oC84	tl4	oC16	hP4 >223GP
ptr/GPa	2.4	4.2	4.3	12	72	

Different types of **ordered HGL** host-guest lattices are represented by



**disordered HGD** host-guest lattices are represented by



# The Nitrogen Group

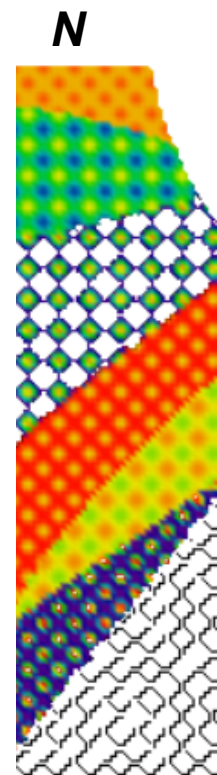
<b>N</b>	gas	hP4	cP16	tP32	hR8	oP16	cl8	>120GPa
ptr/GPa	2.6	4.7	8	18	50	115	cP8 tP4	
<b>P</b>	oC8	hR2	cP1	ML	hP1	cl2	>280GPa	
ptr/GPa	5	10	107	137	258			
<b>As</b>	hR2	cP1	HGL	cl2	>122GPa			
ptr/GPa	25	48	97					
<b>Sb</b>	hR2	HGL	HGL	cl2	>65GPa			
ptr/GPa	8.2	9.0	28					
<b>Bi</b>	hR2	mC4	HGL	cl2	>220GPa			
ptr/GPa	2.5	2.7	7.7	oC16HT				

1TPa

100GPa

10GPa

1GPa



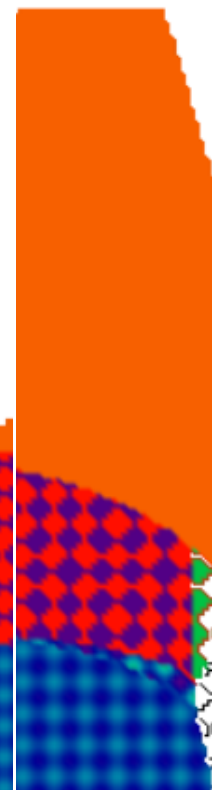
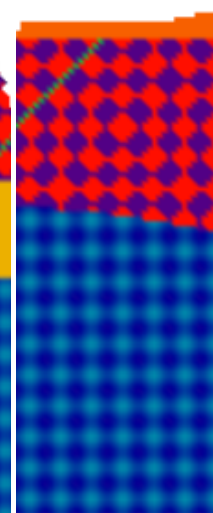
200 - 500K



**As**

**Sb**

**Bi**



1TPa

100GPa

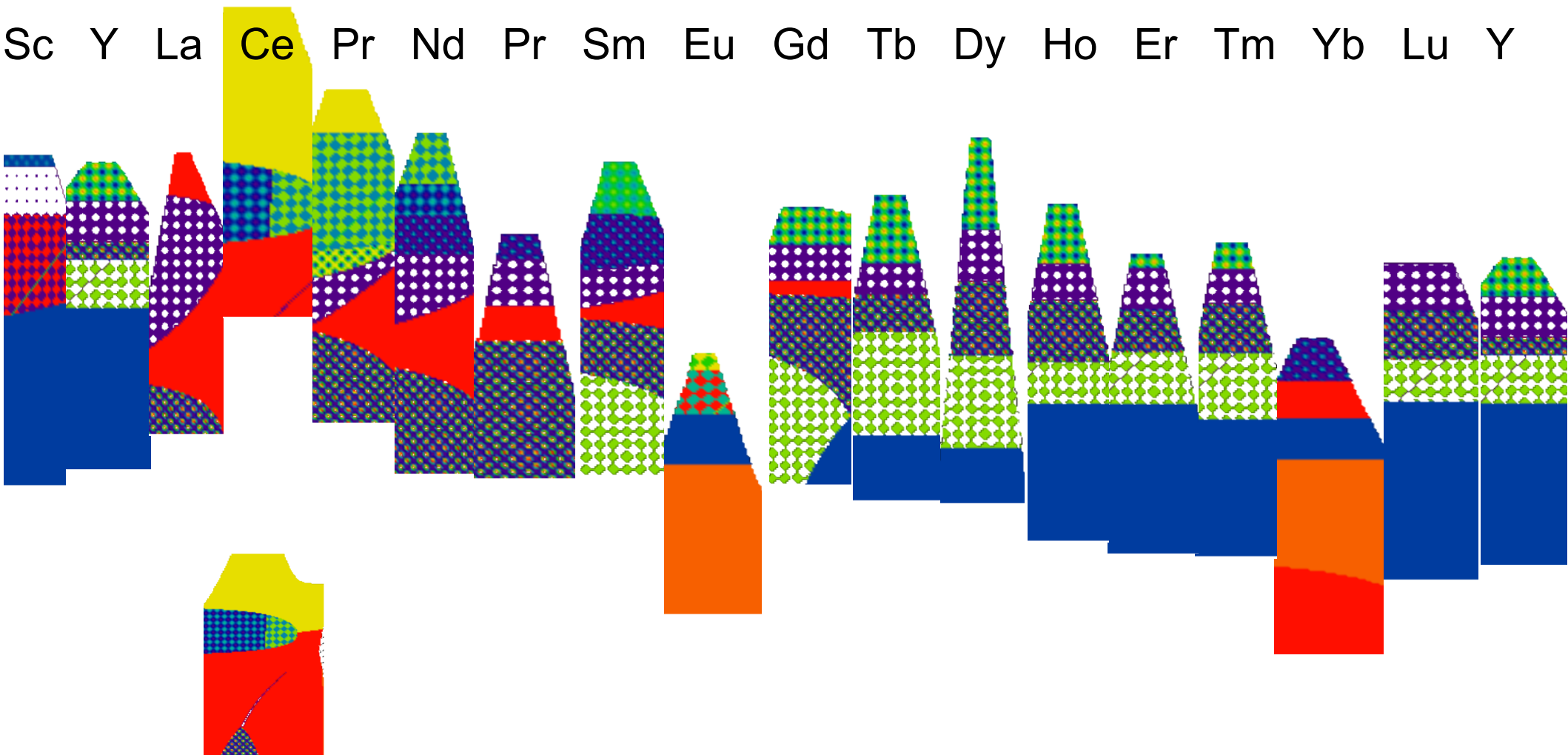
10GPa

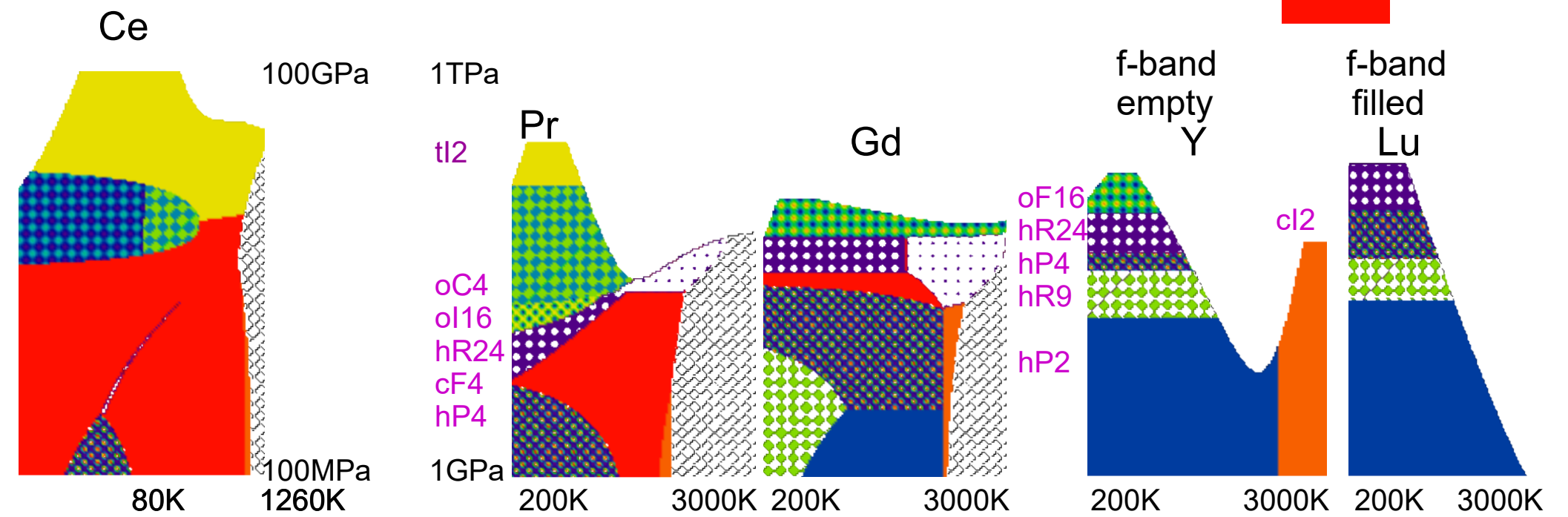
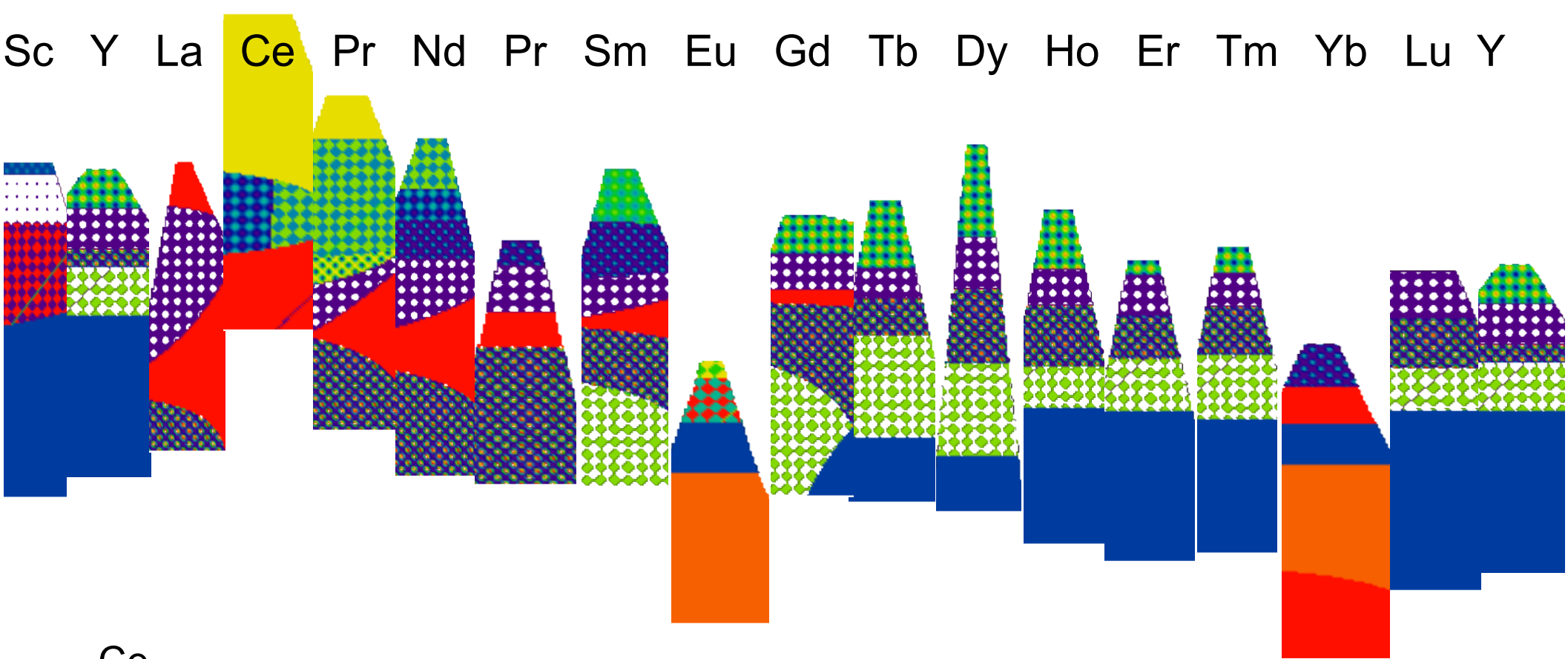
1GPa

**La** hP4—cF4—hR8—cF4—>70GPa  
 ptr/GPa 2 7 58 **d2HT**  
**Ce** hP4—cF4—cF4—oC4—hI2—>208GPa  
 ptr/GPa .2 .67 3.9 13 **d2HT**  
**Pr** hP4—cF4—hR8—???—oC4—>60GPa  
 ptr/GPa 4 6.2 10 20 **d2HT**  
**Nd** hP4—cF4—hR8—hP3—mC4—oC4—>70GPa  
 ptr/GPa 5 15 40 75 113 **d2HT**  
**Pm** hP4—cF4—hR8—hP3—>60GPa  
 ptr/GPa 10 18 40  
**Sm** hR3—hP4—cF4—hP3—hI2—mC4—>70GPa  
 ptr/GPa 5 15 75 105 **d2HT**  
**Eu** cI2—hP2—MI—hI2—>12GPa  
 ptr/GPa 13 31.5

**Gd** hP2—hR3—hP4—cF4—hR8—mI2—>235GPa  
 ptr/GPa 3.2 7 26 46 61 **d2HT**  
**Tb** hP2—hR3—hP4—cF4—hR8—>58GPa  
 ptr/GPa 3 6 29 32  
**Dy** hP2—hR3—hP4—cF4—hR8—>70GPa  
 ptr/GPa 5 9 38 40  
**Ho** hP2—hR3—hP4—hR8—>70GPa  
 ptr/GPa 10 25 70  
**Er** hP2—hR3—hP4—hR8—>70GPa  
 ptr/GPa 12.4 24 67.4  
**Tm** hP2—hR3—hP4—>42GPa  
 ptr/GPa 10 30  
**Yb** hP2—cF4—cI2—hP2—cF4—hP3—>202GPa  
 ptr/GPa 0.1 4 30 53 98  
**Lu** hP2—hR3—hP4—cF4—Lu(IV)—>275GPa  
 ptr/GPa 20 30 88 272

Sc Y La Ce Pr Nd Pr Sm Eu Gd Tb Dy Ho Er Tm Yb Lu Y





## IA

**H** gas---hP4H---->200GPa  
ptr/GPa 5.3 hP4B hP4D

**Li** cl2---cF4-- liquid --oC40----oC24---->116GPa [MN06,VD06,OD09,GG11]  
ptr/GPa 7.5 39 75 90 hR3  
200K: hR1---cl16---oC88-----oC40-----oC24

**Na** cl2---cF4---cl16--oP8\*\*\*--HGL--hP4->215GPa [GD05,MN06,VD06,GL08,LG09,ME09,OD09]  
ptr/GPa 65 104 115 125 180 hP2 mP512HT oP8\*\*\* includes t150 oC120 aP90

**K** cl2---cF4---HGL---oP8---tl4---oC16---->112GPa [MN06,VD06,OD09]  
ptr/GPa 11.6 20 54 90 96

**Rb** cl2---cF4---oC52---HGL---tl4---oC16---->70GPa [MN06,VD06,OD09]  
ptr/GPa 7 13 17 20 48

**Cs** cl2---cF4---oC84-----tl4---oC16---hP4-->223GPa [MD03,MN06,VD06,OD09]  
ptr/GPa 2.4 4.2 4.3 12 72

## IIA

[MN06, VD06, OD09]

**Be** hP2----->171GPa  
ptr/GPa

**Mg** hP2-----cl2----->58GPa [YK12]  
ptr/GPa 50

**Ca** cF4--cl2--cP1---tP8---oC8---mC6---HGL->216GPa [YN05,VD06,NY07, FN08,MW10,NS10]  
ptr/GPa 20 32 113 139 158 210 oC2 tl4 [OM10,SN11,TD12,LD12, FN13,VD14]

**Sr** cF4 ---cl2---tl4---ml12---HGL---->117GPa [BA05]  
ptr/GPa 3.5 27 35 49

**Ba** cl2-----hP2-----HGL----hP2----->105GPa  
ptr/GPa 5.5 12 45 cF4HT

## IIIA

[MN06,VD06]

**B** hR106-----hR12-----oP28---oC8---->250GPa [BT04,OC09,OS11]  
ptr/GPa 3 tP192HT 18 89

**Al** cF4-----hP2----> 333GPa [AN06]  
ptr/GPa 217

**Ga** oC8---liquid---tl2---oC104---hR6---tl2---cF4--->150GPa [DM04,MN06]  
ptr/GPa 0 2.0 2.8 tI2HT 10.5 14 120

**In** tl2---oF4---->93GPa  
ptr/GPa 45

**Tl** hP2---cF4---->68GPa  
ptr/GPa 3.7 cl2HT

## IVA

**C** hP4----cF8---->275GPa  
ptr/GPa 3

**Si** cF8---tl4---ol4---hP1---oC16---hP2---cF4---->250GPa  
ptr/GPa 12 13 16 38 42 80

**Ge** cF8---tl4---ol4---hP1---oC16---hP2---->180GPa  
ptr/GPa 11 75 85 102 160

**Sn** cF8---tl4---tl2---ol2---cl2---hP2---->194GPa [SG11,SB13]  
ptr/GPa 0 9.2 32 70 157

**Pb** cF4---hP2---cl2---->272GPa  
ptr/GPa 13 127

## VA

**N** gas---hP4---cP16---tP32---hR8---oP16---cl8-->120GPa [EG04,GG07,SL09,MO09]  
ptr/GPa 2.6 4.7 8 18 50 115 cP8 tP4

**P** oC8---hR2---cP1---ML---hP1---cl2---->280GPa [VD06,MA08]  
ptr/GPa 5 10 107 137 258

**As** hR2---cP1---HGL---cl2---->122GPa [DM04,VD06,MN06]  
ptr/GPa 25 48 97

**Sb** hR2---HGL---HGL---cl2---->65GPa [MD07,OD09]  
ptr/GPa 8.2 9.0 28

**Bi** hR2---mC4---HGL---cl2---->220GPa [MD07,OD09]  
ptr/GPa 2.5 2.7 7.7 oC16HT

## VIA

**O** gas---hR2---oF8---mC16---mC16---->132GPa [FA06,LW06,WD09,KL10]  
ptr/GPa 6 8 9 96 mC4 cP16

**S** oF128---hP9---tl16---al6---MG---hR1---->160GPa [DG05abc]  
ptr/GPa 1 29 75 90 155 [MN06,HF06,OD10]

**Se** hP3---???---al6---ML---hR1---cl2---->150GPa [DG05ab,MN06,HF06,OD10]  
ptr/GPa 14 23 35 80 140 tl16HT

**Te** hP3---???---al6---ML---cl2---->75GPa [MH04,DG05,MN06,HF06,OD10]  
ptr/GPa 4.0 4.3 4.5 29 hR1HT

**Po** cP1---->0GPa  
ptr/GPa hR1HT

## VIIA

**F** gas---cP16---mC4---->10GPa  
ptr/GPa 2.5 ~20

**Cl** gas---oC8---->45GPa  
ptr/GPa 1

**Br** liquid---oC8---ML---ol2---->125GPa [KH05]  
ptr/GPa 0.2 83 120

**I** oC8---ML---ol2---tl2---cF4---->276GPa  
ptr/GPa 23 25 43 53

## VIIIA

**He** gas---hP2---->58GPa [LL93]  
ptr/GPa 12 cl2 cF4

**Ne** gas---cF4---->110GPa  
ptr/GPa 5.1

**Ar** gas---cF4---->77GPa  
ptr/GPa 1.1

**Kr** gas---cF4---hP2---->50GPa  
ptr/GPa 0.93 20

**Xe** gas---cF4---hP2---->236GPa  
ptr/GPa 0.69 20

**IB****Cu** cF4---->236GPa

ptr/GPa

**Ag** cF4---->236GPa

ptr/GPa

**Au** cF4----hP2---->275GPa

ptr/GPa 230

[DDC07]

**IIB****Zn** hP2---->126GPa

ptr/GPa

**Cd** hP2---->75GPa

ptr/GPa

**Hg** hR1----tl2----oP4----hP2---->67GPa

ptr/GPa 4 12 36

[MN08]

**IIIB****Sc** hP2----HGL----Sc(III)----Sc(IV)----hP6---->297GPa

ptr/GPa 20 104 140 240

**Y** hP2----hR3----hP4----cF4---->75GPa

ptr/GPa 13 26 46

[MN08]

**IVB****Ti** hP2----hP3---->oC4?----oC4?---->220GPa

ptr/GPa 7 116 140 cl2HT

**Zr** hP2----hP3----cl2---->75GPa

ptr/GPa 4 32 cl2HT

**Hf** hP2----hP3----cl2---->252GPa

ptr/GPa 34 71 cl2HT

[MN08]

[VS01\_AKB01\_JJ02]

**VB****V** cl2---->160GPa

ptr/GPa

**Nb** cl2---->150GPa

ptr/GPa

**Ta** cl2---->174GPa

ptr/GPa

**VIB****Cr** cl2---->10GPa

ptr/GPa

**Mo** cl2---->272GPa

ptr/GPa

**W** cl2---->364GPa

ptr/GPa

**VII B****Mn** cl58---->190GPa

ptr/GPa cP20HT

**Tc** hP2---->10GPa

ptr/GPa

**Re** hP2---->251GPa

ptr/GPa

**VIII B**

**Fe** cl2----hP2---->305GPa  
ptr/GPa 11 cF4HT

**Ru** hP2---->25GPa  
ptr/GPa

**Os** hP2---->75GPa  
ptr/GPa

[MN06]

[AD13]

**IX B**

**Co** hP2-----cF4---->202GPa  
ptr/GPa 120

**Rh** cl2---->56GPa  
ptr/GPa

**Ir** cF4-----hP14---->65GPa  
ptr/GPa 59

[MN06]

**XB**

**Ni** cF4---->236GPa  
ptr/GPa

**Pd** cF4---->80GPa  
ptr/GPa

**Pt** cF4---->300GPa  
ptr/GPa

**Ln**

**La** hP4---cF4---hR8---cF4---->70GPa  
ptr/GPa 2 7 58 cl2HT

**Ce** hP4---cF4---cF4---oC4---tl2---->208GPa  
ptr/GPa .2 .67 3.9 13 cl2HT

**Pr** hP4---cF4---hR8---???---oC4---->60GPa  
ptr/GPa 4 6.2 10 20 cl2HT

**Nd** hP4---cF4---hR8---hP3---mC4---oC4---->70GPa  
ptr/GPa 5 15 40 75 113 cl2HT

**Pm** hP4---cF4---hR8---hP3---->60GPa  
ptr/GPa 10 18 40

**Sm** hR3---hP4---cF4---hR8---hP3---tl2---mC4---->70GPa  
ptr/GPa 5 15 20 37 75 105 cl2HT

**Eu** cl2---hP2---ML---???---->92GPa  
ptr/GPa 13 31.5 37 [MK11, HL12]

**Gd** hP2---hR3---hP4---cF4---hR8---ml2---->235GPa  
ptr/GPa 3.2 7 26 46 61 cl2HT

**Tb** hP2---hR3---hP4---cF4---hR8---->58GPa  
ptr/GPa 3 6 29 32 [TM13]

**Dy** hP2---hR3---hP4---cF4---hR8---->70GPa  
ptr/GPa 5 9 38 40

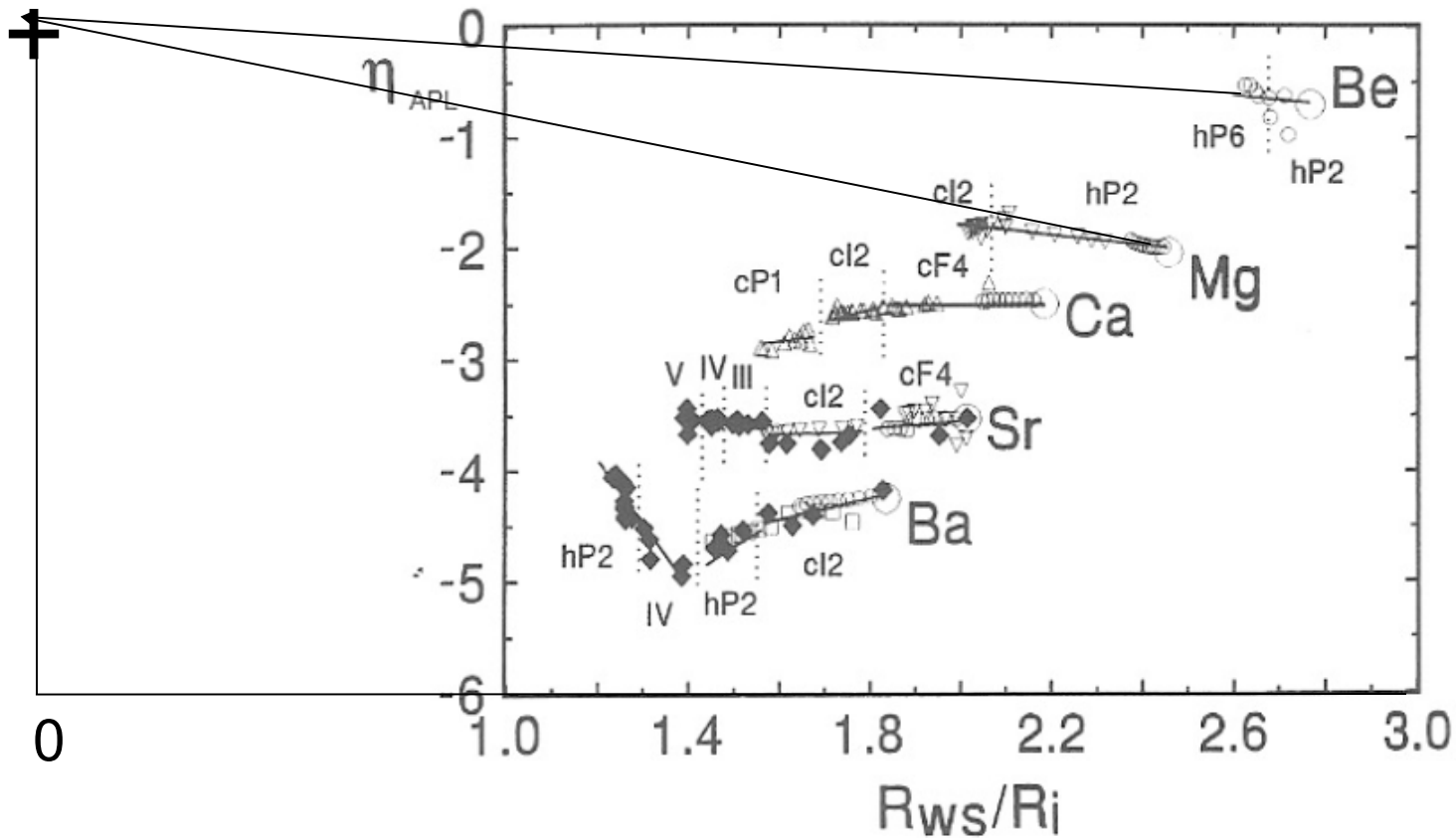
**Ho** hP2---hR3---hP4-----hR8---->70GPa  
ptr/GPa 10 25 70

**Er** hP2---hR3---hP4-----hR8---->70GPa  
ptr/GPa 12.4 24 67.4

**Tm** hP2---hR3---hP4---->42GPa  
ptr/GPa 10 30

**Yb** hP2---cF4---cl2---hP2---cF4---hP3---->202GPa  
ptr/GPa 0.1 4 30 53 98

**Lu** hP2---hR3---hP4---cF4---Lu(IV)---->275GPa  
ptr/GPa 20 30 88 272



**Fig. 17.** APL linearization for the EOS data of the alkaline earth metals plotted versus the scale Wigner-Seitz radius ratio  $R_{WS}/R_i$  discussed in the text using EOS data from the different sources listed in the original work [7, 37].

"Structural Systematics and Equations of States for the Alkaline Earth Metals under Pressure"  
M. Winzenick and W.B. Holzapfel, p. 384-386 in "High Pressure Science and Technology" ed.  
W. Trzeciakowski, World Scientific, Singapore (1996)

"Physics of Solids under Strong Compression"  
W.B. Holzapfel, Rep. Prog. Phys. 59, 29-90 (1996)

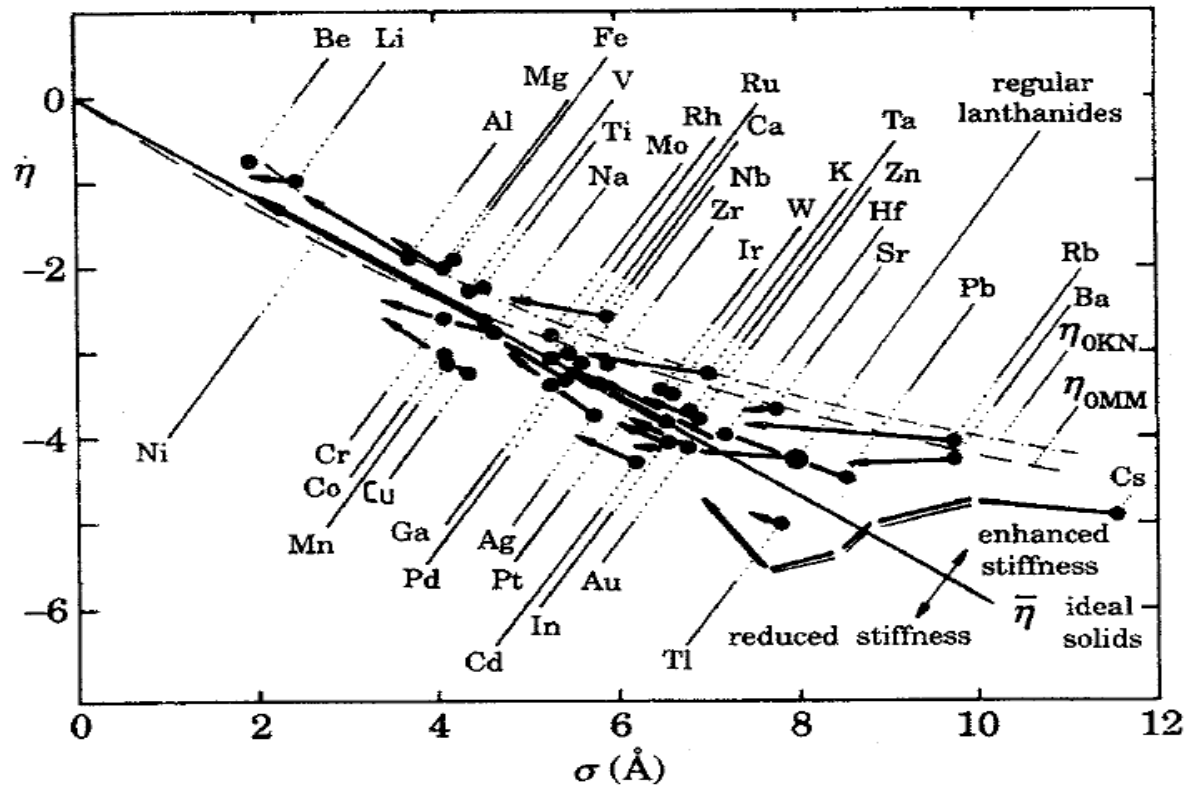


Fig. 1. - EOS data [6, 11-30] for the metallic elements represented in  $\eta = \ln(p/p_{FG}) - \ln(1 - (\sigma/\sigma_0))$  vs.  $\sigma$  plots with  $\sigma = (ZV \cdot 3/(4\pi))^{1/3}$ . The linear correlation  $\eta$  vs.  $\sigma$ , which defines the «ideal» solid behaviour is also compared with earlier correlations between ambient pressure values of  $K_0$  and  $\sigma_0$  discussed by McMillan [6] and Knopoff [9] and marked with  $\eta_{0MM}$  and  $\eta_{0KM}$ , respectively. Anomalies for the high-pressure phases of Cs are marked by twinned lines.

"Equations of State for Ideal and Real Solids under Strong Compression"  
 W.B. Holzapfel, Europhys. Lett. 16, 67-72 (1991)

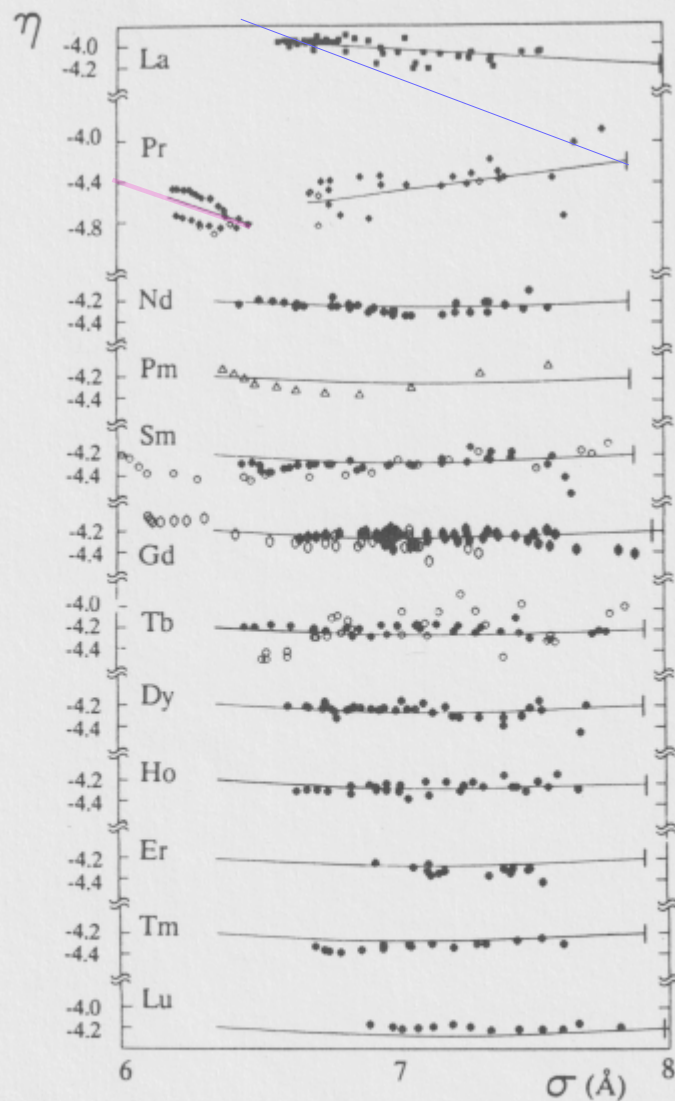


FIG. 16. EOS data for the “regular” lanthanides in  $\eta$ - $\sigma$  representation with  $\eta = \ln(p/p_{FG}) - \ln(1 - \sigma/\sigma_0)$  using Thomas-Fermi scaling for the atomic length parameter  $\sigma$  and the Fermi gas pressure  $p_{FG}$ . Open symbols refer to recent data from the literature (Refs. 14–23) for this extended pressure range.

"Atomic Volumes for Rare Earth Metals under Pressure to 40 GPa and Above"  
W. A. Grosshans and W.B. Holzapfel,  
Phys. Rev. B 45, 5171-5178 (1992)

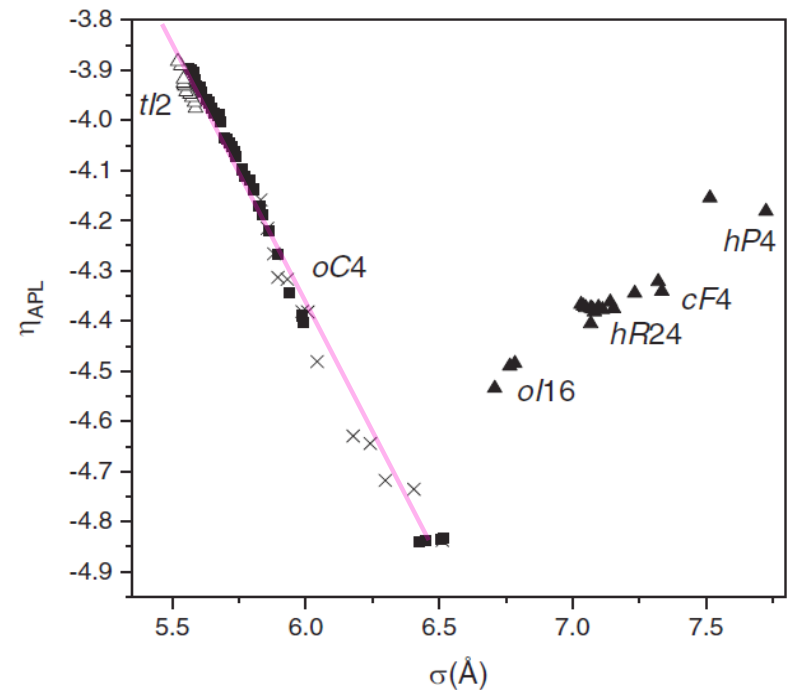


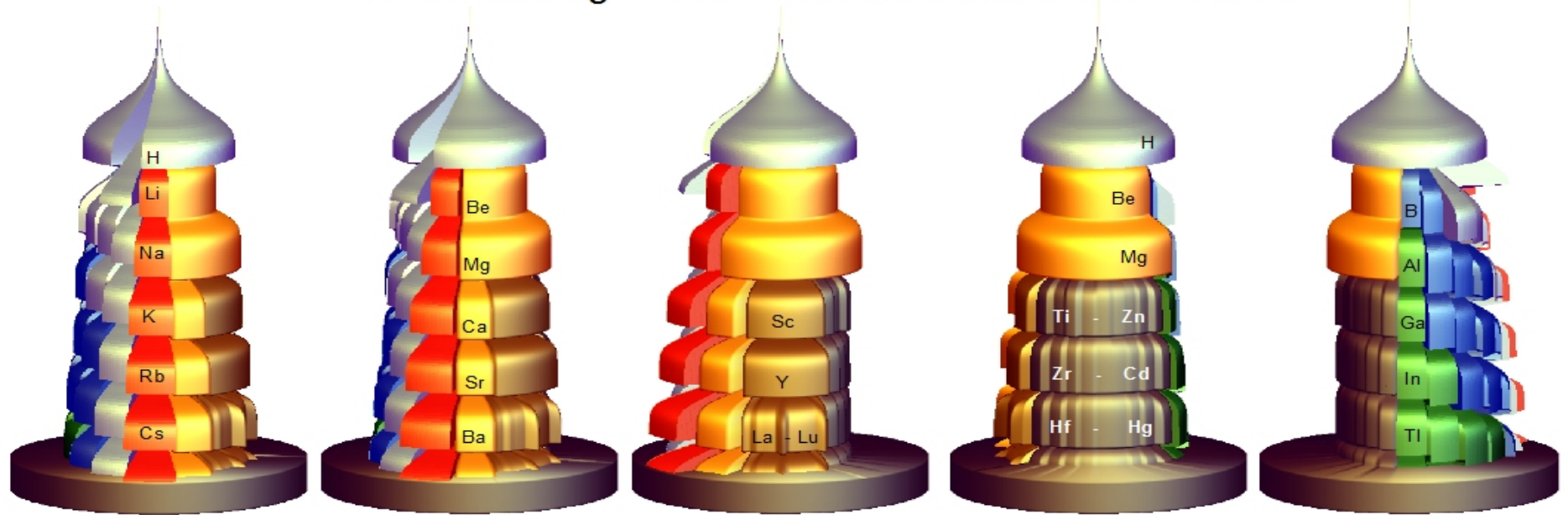
FIG. 7. Linearization of the compression of Pr shown in the form of an  $\eta_{APL}$ - $\sigma$  plot, where  $\sigma = \sigma_0 x$ . The data from the lower-pressure phases (*hP4*, *cF4*, *hR24*, and *oI16*) are plotted with filled triangles, while our data from *oC4*-Pr are plotted with filled squares and the *oC4*-Pr data from the study of Chesnut and Vohra [29] are shown as crosses. For clarity, the current data for *tI2*-Pr are plotted with unfilled triangles.

— Simple 1. order AP1-EOS

High-pressure structure of praseodymium revisited:  
In search of a uniform structural phase sequence for  
the lanthanide elements

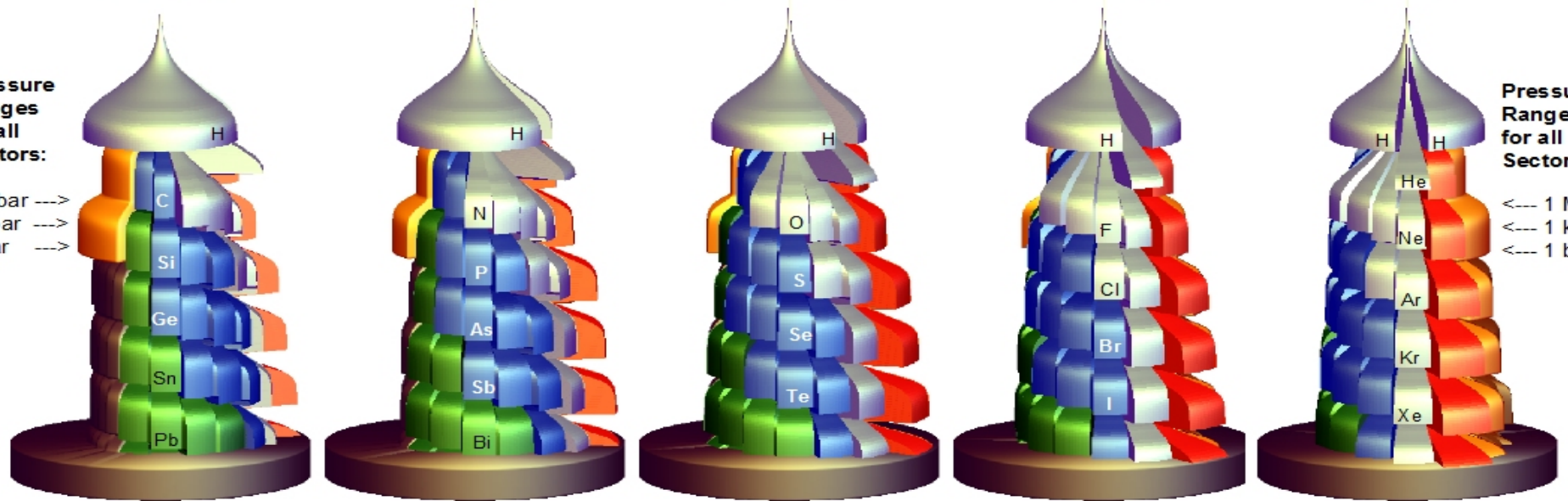
S.E. Finnegan, M.G. Stevenson, E.J. Pace, C.V.  
Storm, J.D. McHardy, M.I. McMahon, S.G.  
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Phys. Rev. B **105**, 174101 (2022)

# The fascinating World of the Elements under Pressure



Pressure Ranges for all Sectors:

1 Mbar ---->  
1 kbar ---->  
1 bar ---->



Pressure Ranges for all Sectors:

<--- 1 Mbar  
<--- 1 kbar  
<--- 1 bar

Model for a sculpture to be decorated with the colourful phase diagrams.  
In honour of Aristoteles (†322 B.C.) and Galilei (†1642 A.D.) founders of modern science proclaiming:

*Measure all you can measure  
and what you can't, make it measurable !*



## United States of the Elements – USE

At the beginning there was a big bang, which generated the Hydrogen and little Helium, that is true, was in this early stardust brew. Some dust condensed by gravity and stars appeared with density just high enough for the ignition of nuclear fusion with the provision for many more new elements, but these were not the final ends! The heavy ones had still to wait for supernovae, which came late as star explosions, an event, which is considered as the end of all the element creation, we only miss still the equation explaining how the numbers came to fit all this! This is the game! With all the elements now here, some order makes this world more clear. Wise man produced a Periodic Table with 8 Main Groups which do enable to show some common properties in 8 columns, but there are these remaining elements between with 10 at first and then 14, Transition Metals, 3 rows of 10, and Lanthanoides, with 14 then. The Actinoides, we will not need, they are unstable, well indeed!- And still more order is present, if you look carefully at the end! You can first see right on the top the Hydrogen and if you stop, you see for Helium a slit just wide enough to give a fit for the Rare Gases as column on top with our Helium. The other columns broad and wide include the Main Groups not so tide as all the lower sections squeezed between for the Transition Metals in this scheme. And in the column number 3 the Rare Earth Metals you can see. The chemists now look for reactions. The physicists see other actions! They change the temperature and pressure and keep in mind, they have to measure the properties in all these states and some results lead to debates before one reaches the consensus on these results, which are immense in these domains, which reach as far, as the interior of some star! Much more ideas have been cast into this figure, but at last a book can only give to you for this artwork the final clue!

# Colorful systematic in the phase-diagrams of the elements

A simple guide to the world of high pressure

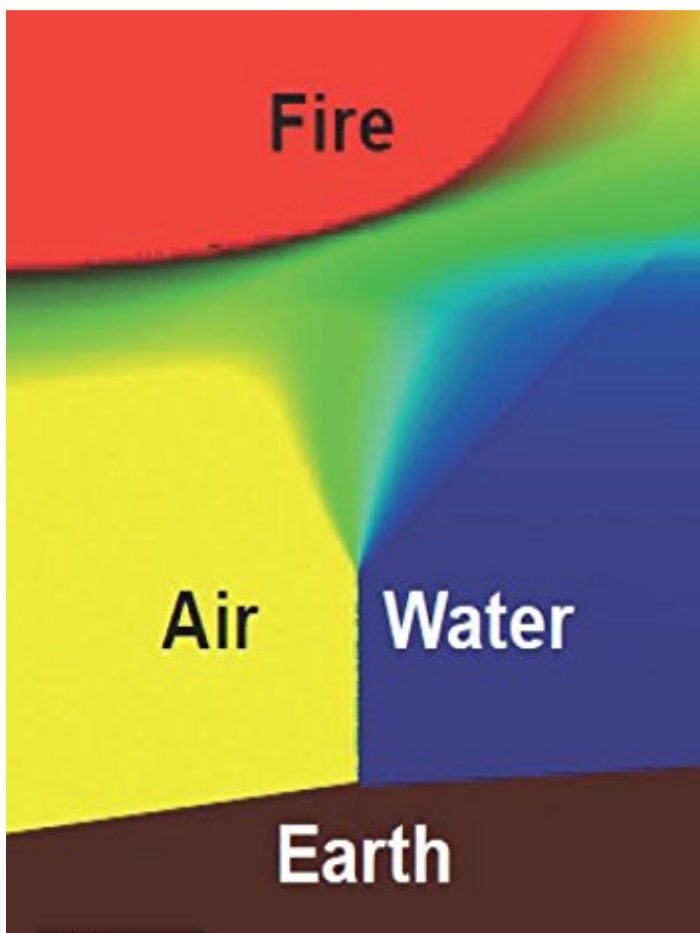
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Intention: stimulate young scientists to look at structural changes in matter  
in wide fields of pressure and temperature

4 essentia

4 classical elements

4 forms of being for matter



states of matter  
in wide ranges  
of  
temperature  
and  
pressure

plasma

gas

liquid

solid

