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Preface

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1
A Thermodynamic Formulation of Social Science

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A Thermodynamic Formulation of Social Science

Abstract

The thermodynamic formulation of social laws is based on the law of statistics under constraints, the Lagrange principle. This laws is called "free energy principle" in physics and has been applied successfully to all fields of natural science. In social sciences it may be called the "principle of maximum happiness" of societies. The principle may be applied to collective and individual behavior of social, political or religious groups. Opinion formation like US elections may be simulated, as well as the division of the world in hierarchic and democratic countries. The principle applies to data of segregation of Blacks and Whites in the US, to citizen and foreigners in Germany, to political and religious separation in Northern Ireland and Bosnia.

Introduction

Social models have been presented by many authors like TH. SCHELLING, W. WEIDLICH, R. AXELROD, S. SOLOMON, D. STAUFFER, S. GALAM and many others. In this paper the Lagrange principle is presented as a very general approach to social science. Socio-economics deals with large systems, the people of a town, a state, a continent. In large crowds people do not know each other. Few symbols like face, skin, hair, clothing, language indicate race, gender, ethnicity, country or religion. The reactions and interactions in large groups depend only on these little bits of information. And since there is often not enough time, most people will quickly generalize and attribute properties to large groups: American travelers are rich, Japanese workers are busy, Jewish business men are smart, Italian men are charming. These generalizations are the properties of groups in the heads of other people, whether they are true or not. This is an "atomization" of social interactions. Accordingly, the elements of socio- economic systems are not people, but agents with few obvious properties. The behavior of agents is determined by probability under constraints. The Lagrange principle is the basis for the understanding and simulation of socio-economic interactions.

1.1 Probability

Calculations of probability have their roots in playing games and taking chances at gambling. Fig. 1.1 shows a simple nail board, a predecessor of modern flipper machines. Each time a ball hits a nail, the ball has an even chance to fall to the left or to the right side. Each ball has its own course. But at the end all balls fall into different boxes and form a curve shaped like a bell, the normal distribution.

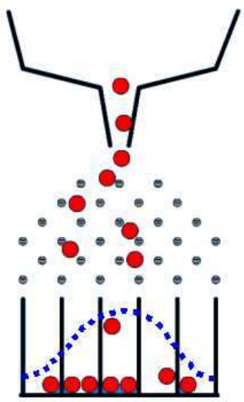

Fig. 1.1 Galton´s nail board: A number of balls pass through a nail board and either fall to the left or right side. The course of each single ball is unpredictable and chaotic, but the final distribution in the five boxes leads to a bell shaped normal distribution.

This normal distribution can be predicted more prescisely the more balls are used. This is a simple but surprising result. The course of each single ball cannot be predicted by mechanics due to the six bifurcations, which lead to chaotic movement of the balls. But still the sum of all courses is a well defined normal distribution. This bell shaped distribution demonstrates that there is order in chaos. The same may be expected for a society of millions of chaotic individuals, who cannot be predicted individually but only as a society.

1.1.1 Normal Distribution

The distribution of N cars parked on the two sides of a street is calculated from the laws of combinations: the probability of N_l cars are parking on the left side and N_r cars on the right side of the street is given by

$$P(N_l; N_r) = \frac{N!}{N_l! N_r!} q^{N_l} (1-q)^{N-N_l} \tag{1.1}$$

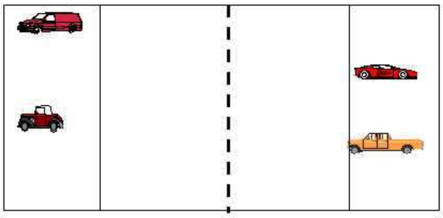

Fig. 1.2 The distribution of one half of the cars on each side of the street is most probable. According to Eq.(1.1) the probability for $N_l = 2$ and $N_r = 2$ is given by $P(2;2) = 6:16$ or $37,5\%$.

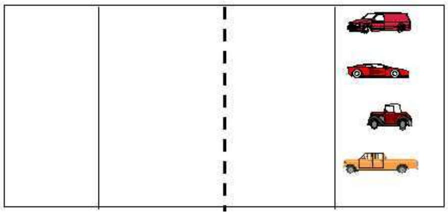

Fig. 1.3 The distribution of all cars on one side and none on the other side has always the least probability. For $N_l = 0$ and $N_r = 4$ we find the probability according to Eq.(1.1): $P(0;4) = 1:16$ or $6,25\%$.

In fig. 1.2 the cars are evenly parked on both sides of the street. The probability of this even distribution has always the highest probability. According to Eq.(1.1) we find $P(2;2) = 6:2^4 = 6/16$ or $37,5\%$.
In fig. 1.3 four cars are all parked on one side of the street and none on the other. The distribution on one side has always the lowest probability. For equal space on both sides, $q = 1/2$ in Eq.(1.1) we find $P(0;4) = 1:2^4 = 1/16$ or $6,25\%$.

1.1.2 Constraints

In fig. 1.4 the "no parking" sign on the left side forces the cars to park on the right side, only. The "no parking" sign is a constraint, that enforces the least probable distribution of cars, all on one side. A constraint or law generally enforces a very improbable distribution, which would otherwise not be observed.
In fig. 1.5 we find one individual driver ignoring the collective "no parking" sign. What is the probability of this unlawful distribution? Figs. 1.4 and 1.5 are closely connected to the problem of behavior of socio-economic agents. People often act individually against collectively accepted rules, trends, opinions or laws. What is the probability of this distribution of cars on the road? The problem may be solved by looking at the laws of probability with constraints.

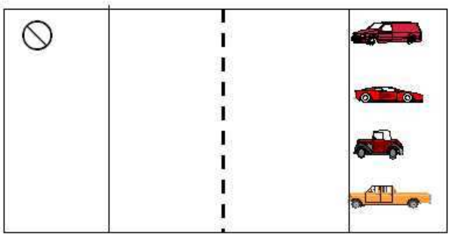

Fig. 1.4 The “no parking” sign enforces the least probable distribution of cars on one side of the street.

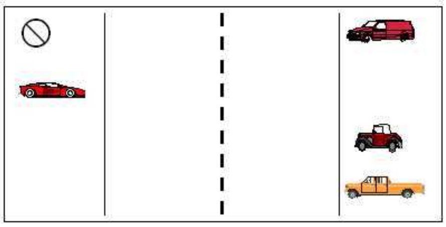

Fig. 1.5 One driver is ignoring the “no parking” sign.

1.1.3 Probability with constraints (Lagrange principle)

Probability with constraints may be calculated by a principle introduced by JOSEPH DE LAGRANGE (1736-1813),

$$L = E + T \ln P \rightarrow \text{maximum!} \tag{1.2}$$

L is the Lagrange function of a system of N interacting elements. P is the combinatorial probability of the distribution of the elements, Eq.(1.1), and $\ln P$ is called entropy. The function E stands for a constraint of the system. In fig. 1.5 the constraint is given by the “no parking” sign $\oslash$ on the left side of the road and the corresponding fine for violation. T is the Lagrange parameter. The maximum of L is called equilibrium of the system. The meaning of the functions L, E, T and $\ln P$ varies with the system. The dimension of the functions depends on the constraint E, $\ln P$ is without dimension:

1. In atomic systems the constraint E is the energy, T is the mean kinetic energy or temperature, $(-L)$ is the free energy. $\ln P$ is the entropy. The atomic system is stable at the maximum of the free energy.

2. Social systems are governed by collective laws (E), $\ln P$ represents the individual behavior. T is the tolerance (of individual behavior), L is the common happiness of the agents about the behavior in the society. The social system is stable at the maximum of mutual happiness.

3. In political systems emotions (E) between different groups are important constraints, $\ln P$ represents the difference of groups. T is the tolerance (of other groups). L is the happiness of the groups about the structure of the society. The political system is stable at the maximum of mutual happiness.

4. In economic systems the constraint E is the capital, $\ln P$ represents the individual chance. T is the mean capital or standard of living, L is the sum of everybody's self interest. The economic system is stable, if everybody's self interest is at maximum.

The functions L, E, T, P in the Lagrange principle may be interpreted in very general terms. The function E is a law, opinion or decision that acts as a constraint and requires a specific order of the system. The entropy function $\ln P$, on the other hand, is a measure of disorder for the law (E). The parameter T is something like the tolerance of disorder of the law (E). A typical example of order and disorder is a children's room. The order of toys is usually defined by the parents. But due to the high probability of disorder a children's room will be chaotic, if the parents tolerate just a little bit of disorder. The Lagrange parameter T acts as a switch between order and disorder.
The Lagrange principle (1.2) may be interpreted in many different ways:

Table 1.1

Lagrange:	$L = E$	$+$	$T \ln P$	$\rightarrow$maximum!
Probability:	$L =$ improbable	$+$	T probable	$\rightarrow$maximum!
Physics:	$L =$ energy	$+$	T entropy	$\rightarrow$maximum!
Law:	$L =$ order	$+$	T disorder	$\rightarrow$maximum!
Structure:	$L =$ simple	$+$	T complex	$\rightarrow$maximum!
Behavior:	$L =$ collective	$+$	T individual	$\rightarrow$maximum!
Appearance:	$L =$ uniform	$+$	T variety	$\rightarrow$maximum!
Organization:	$L =$ organized	$+$	T disorganized	$\rightarrow$maximum!
Behavior:	$L =$ planned	$+$	T spontaneous	$\rightarrow$maximum!
Welfare:	$L =$ fair	$+$	T unfair	$\rightarrow$maximum!
Ethics:	$L =$ good	$+$	T bad	$\rightarrow$maximum!
Law:	$L =$ right	$+$	T wrong	$\rightarrow$maximum!
Art:	$L =$ beautiful	$+$	T ugly	$\rightarrow$maximum!
Health:	$L =$ healthy	$+$	T sick	$\rightarrow$maximum!
Biology:	$L =$ living	$+$	T dead	$\rightarrow$maximum!
Engineering:	$L =$ correct	$+$	T defect	$\rightarrow$maximum!
Chemistry:	$L =$ cohesion	$+$	T entropy	$\rightarrow$maximum!
Society:	$L =$ bonds	$+$	T freedom	$\rightarrow$maximum!
Economics:	$L =$ ratio	$+$	T chance	$\rightarrow$maximum!

Apparently, the Lagrange equation is a principle that goes far beyond physics or socio-economics. It is one of the most general principles in all aspects of life.

1.2 Elements of Societies

Many people believe that human behavior cannot be calculated by any model, especially if the model has been adopted from atoms and molecules. Human individuals are much more complex than atoms or molecules!
Indeed, no mathematical equation can model the behaviour of a single person. But a statistical approach to human nature never applies to single persons, only to large crowds. In a large crowd individual properties are lost and groups may be labelled by certain features: In a football stadium the competing parties wear different shirt colours. In the streets foreigners are recognized by their accent. Dress, uniform, language, sex, skin, hair are most obvious labels for unknown people. All other properties remain undiscovered.

1.2.1 Agents

Agents are the elements of social systems. Agents are representatives of persons, as they appear in the mind of other people. Agents are defined by their properties:

$$\text{Agent}: [A; B; G; H; .....]$$

By adding an infinite number of features it would be possible to model a real individual person. But in large crowds people will only notice very few properties of an individual and accordingly, all individuals may be categorized by few properties. These properties may be scalable like tall, fast, strong, beautiful, smart and rich. Scalable properties lead to classes. Not scalable properties are female, black, Muslim, Indian. They lead to groups.

1.2.2 Groups

A group is a crowd of agents with a common not scalable group property. The group property is often linked to a label (A). Women wear different clothes than men, policemen or nurses have their specific uniforms, foreigners have a specific accent, black people stand out in a white society. The group label may be an idea, an idol or a real person, a president of a club or company, the priest of a parish, the leader of a party, the nucleus of a crystal, the α-animal of a herd. A policeman arrests in the name of the state, a priest marries a couple in the name of the Lord, and a judge makes
a decision in the name of the People. In this way it is possible to find a single name for large groups: Europeans, Catholics, Buddhists, British, French, Yankees, Green party, merchants, scientists, workers and tourists. The most obvious labels are: complexion, sex, heritage, nationality, faith, education,

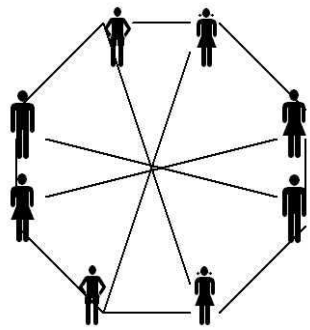

Fig. 1.6 shows a one dimensional (cyclic) arrangement of equivalent agents in a group. All agents have equivalent nearest neighbor bonds. The center (hub) is the nucleus of the group (president of a club, teacher of a school class, policeman in traffic).

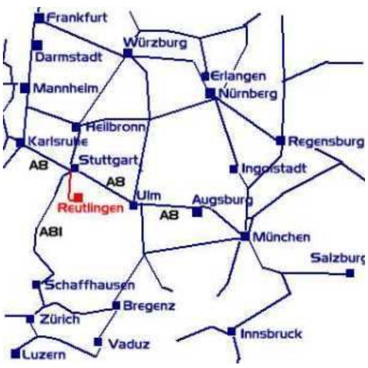

Fig. 1.7 The train network in southern Germany is a two dimensional lattice of equivalent cities. Nearly all cities have equivalent nearest neighbor connections.

profession etc. Within a group all agents are equivalent. Agents of different groups are also equivalent. There is no natural order for British, French or American citizen, for Moslems, Christians or Jews, for blue, yellow or red shirts of a football team. The loss of individuality leads to the "atomization" of societies. Groups in societies correspond to components in atomic systems. Agents may have several properties at the same time. Bur at one instant an agent will generally show only one specific group property:

Example: *A policemen may be married, British and wearing a yellow shirt. But within one group only one property is valid, all other properties are turned off: in his job he is policeman, at home he is a husband, as a voter he is British and on the football field he wears the yellow shirt of his team.*

The size of a group is determined by the number (N_A) of agents with the same group property (A). In social models the size of a group should be larger than

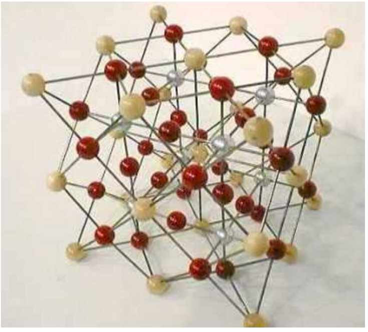

Fig. 1.8 A crystal is a three dimensional lattice of equivalent atoms. All atoms have equivalent nearest (and second nearest) neighbor bonds.

one hundred in order to apply the probability calculations with reasonable accuracy.

1.2.3 Interactions

With our subjective impressions of people we develop emotions to specific group labels. The uniform of a policeman in a dark street makes us feel secure, the white dress of a doctor raises hope for the sick, the robe of a judge indicates justice, the suit of businessmen honesty. People of one country will be bound to their heritage, people of one faith will gather in a common church service, scientists will meet at conferences to talk about their field. The group interaction (E_A) is positive, agents of one group are attracted to each other. The solidarity of a group will grow with the number of common labels of the agents. A famous example ist the WASP population (White, Anglo-Saxon, Protestant) in the U.S.A., which feels closely related and tries to maintain power in the country.
The attractive force leads to a specific structure of groups. Fig. 1.6 shows a cyclic linear chain and represents a group of equivalent agents with positive nearest neighbour bonds. All agents are connected to the centre (hub), the representative (king, president, head) of the group. Fig. 1.7 shows the train system in southern Germany as a two dimensional network of equivalent cities. There are only nearest neighbours connections. (The hub is not shown in the track system, as the head office is connected to the stations by telephone.) A similar network is the system of main highways in most countries, connecting equivalent big cities. All cities again are hubs of local road systems.
In fig. 1.8 the crystal model shows the three dimensional network of equivalent atoms. Only nearest neighbours are shown, but second and higher neighbour interactions are also are present. (The hub of the crystal is the electromagnetic field, which is again not visible). The stable structure of a group

of agents in socio-economic systems is determined by the maximum of the Lagrange function (1.2).

1.2.4 Classes

A class is a crowd of agents with a common scalable group property. The class property is linked to the Lagrange parameter (T) of the Lagrange function (1.2):

1. In atomic systems the constraints are cohesive energies (E). The Lagrange parameter is a mean energy per atom or temperature, $T = E/N$. This leads to the scalable property "warm".

2. In social systems the constraints are given by emotions (E). Accordingly, the Lagrange parameter tolerance of disorder (T) has also an emotional dimension and leads to the scalable property "tolerant".

3. In economic systems capital is a constraint (E), the Lagrange parameter (T) is a market index, a mean capital per capita or standard of living, $T = E/N$. This leads to the scalable property "rich".

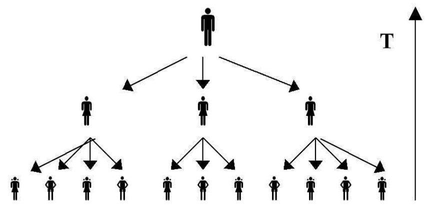

Fig. 1.9 Vertical hierarchic structure of classes in different societies: Pope, priests, laymen in the Catholic church; king, nobility, peasants in medieval times; general, officers, soldiers in the army; director, managers, workers of a company. Agents in the same class form a group with the same value of (T). Agents in different classes are at different values of (T). The communications and orders are only one directional, from the boss to the subordinates. In addition to the arrows of orders the hub has ties of information to all subordinates.

Each system has a specific constraint (E), the Lagrange parameter $T = E/N$, and the corresponding scalable property. Other examples for class properties are tall, fast, strong or beautiful. Agents with the same scalable property are part of the same class. Within a class all agents are nearly equivalent. Members of different classes are not equivalent and may be lined up in a "vertical way". Fig. 1.9 shows the vertical structure of classes in different societies: Pope, priests, laymen in the Catholic church; king, nobility, peasants in medieval times; general, officers, soldiers in the army; director, managers, work-

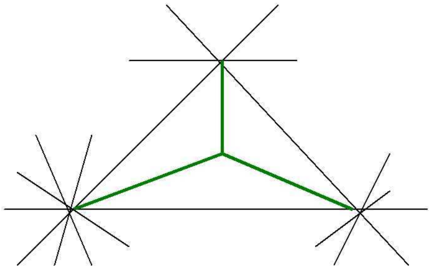

Fig. 1.10 Top view on a hierarchic structure according to fig. 1.9: three local hubs are connected to the centre hub (top). This network corresponds to international telephone networks with national hubs, international airline cooperation, national computer networks, organization network of companies, schools, churches etc.

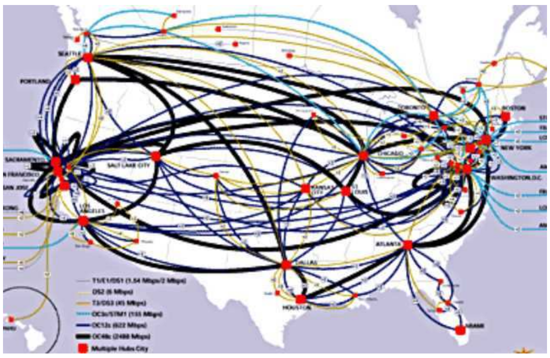

Fig. 1.11 Internet structure of UUNET, one of the world's leading Internet networks. - UUNET, an MCI WorldCom Company.

ers of a company; professor, assistants, students in university; adults, youth, children in the family.

The class properties are the constraints (E) of society: Agents in the same class form a group with nearly the same value of (T). Agents in different classes are at different values of (T), this leads to a hierarchic structure. Agents of one class are linked to a person of a higher class, like in fig. 1.6 indicating the hub of a group being from a different class. Fig. 1.10 shows the hierarchic structure of fig. 1.9 in a view from the top. The hub has connections to all smaller centres. Fig. 1.11 shows the structure of a real network, the Internet (UUNET).

1.2.5 States: Collective vs. Individual

The Lagrange equation (1.2) contains two different states: 1. constraints (E) represent the collective, ordered state and 2. entropy ($S = \ln P$) stands for the individual, chaotic state.

The state of a system depends on the Lagrange parameter (T). This may be observed in atomic systems as well as in socio-economic states:

At low values of T there is little entropy, atoms are in the ordered, collective, solid state, fig. 1.12. In social systems, at work in the army, agents face a low tolerance (T) of individual behaviour and have only one choice (P) like "yes, Sir" or "amen". These agents are in a well ordered, collective and hierarchic state, fig. 1.14. They cannot decide for themselves.

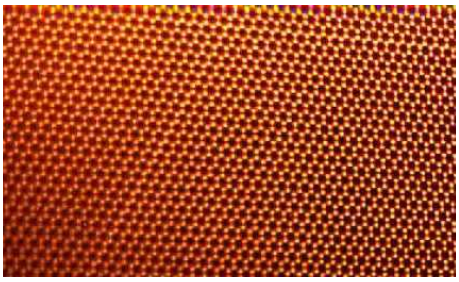

Fig. 1.12 At low temperature (T) atomic systems are in the ordered, single crystal state (model).

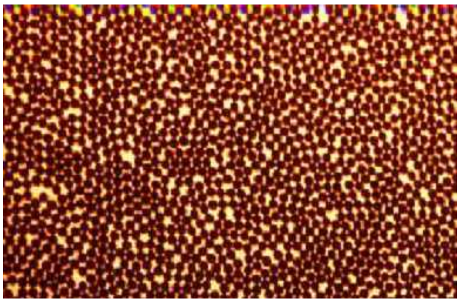

Fig. 1.13 At high temperature (T) atomic systems generally turn into a disordered liquid state (model).

At high values of T there is much entropy, atoms are in the liquid state, fig. 1.13. After work the tolerance for personal decisions is high, agents have many choices (P) like "yes" or "no" or "perhaps". These agents are in the individual, disordered state, fig. 1.15.

States and classes depend on the parameter T. But different classes only show a gradual change with T within the collective state. There are no classes in the individual, disordered state.

Fig. 1.14 At a parade (low tolerance (T) of disorder) soldiers march in perfect order. (With kind permission of the Bundeswehr).

Fig. 1.15 After work (high tolerance (T) of disorder) the soldiers relax in disorder. (With kind permission of the Bundeswehr).

Collective and individual states can be observed in all aspects of social and economic life. In economic systems poor people work in production lines sometimes even in collective movement. Most people with low income work in companies at low positions with no right for personal decisions. Only with higher income employees will have the chance to for more individual actions. In religious systems people in rich countries promote the right of free, individual opinion. In poor countries people react collectively to individual provocations, like in the cartoon crisis between Denmark and the Moslem world.
In normal human development children still have with low ability (T). They live collectively in the family hierarchy, they have to obey their parents, they cannot cross the street at their own will. After puberty young people may already start things on their own, but they still need the family bonds. Only after graduation from school by acquiring the first job people will be grown up and live their individual life.

1.3 Homogenious Societies

In a homogenious society all agents have the same label, all belong to the same group, like Blacks, French or Catholics.

1.3.1 The Three States of Homogeneous Societies

Fig. 1.16 shows the phase diagram of homogeneous (socio-economic or atomic) societies. At low values of the Lagrange parameter (T) a system will be in the ordered state: socio-economic systems will be in the collective state, political systems in the hierarchic state and atomic systems in the ordered solid state. Above a critical value of T_c the systems will change into the disordered state: socio-economic systems will change into the individual state, political systems into the democratic state and atomic systems in the liquid state. At very high values of T the systems turn into another chaotic state: social and political systems become global and atomic states a gas. This is shown in fig. 1.16.

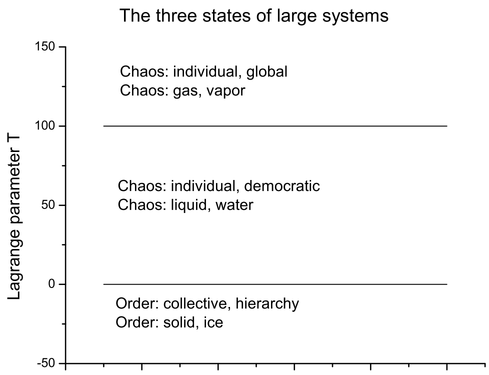

Fig. 1.16 Three states of systems: at low values of the Lagrange parameter (T) the ordered state is stable. Social systems are collective, political systems hierarchic, atomic systems solid. At higher values of T the system turns into the chaotic state: social systems become individual, political systems become democratic, atomic systems become liquid. At very high values of T the system turns into another chaotic state: social and political systems become global and atomic states a gas.

Atomic systems: H_2O
Ordered state (solid): At low temperatures (T) the solid is well ordered and inflexible. Atoms have strong bonds and a low mobility. The solubility or tendency to mix with other atoms is low, we usually find segregation.

Chaotic state (liquid): At higher temperatures (T) the liquid is disordered and flexible. Atoms have weak bonds and a higher mobility. The solubility or tendency to mix with other atoms is higher, we find less segregation.
Global state (gas): At very high temperatures (T) the gas is disordered and flexible. Atoms have no bonds and a very high mobility. The tendency to mix with other atoms is very high, we always have full solubility.

Social systems: Guided Tours

Ordered (collective) state: At low knowledge (T) of the foreign country the tourists follow the guide in a well ordered and collective manner. The rules for exceptions are inflexible. Tourists have strong bonds to their guide and a low mobility of their own. The tendency to mix with people of the foreign country is low, the tourists usually segregate.
Chaotic state (liquid): At better knowledge (T) of the foreign country the tourists follow the guide in a less ordered and less collective manner. The rules for exceptions are flexible. Tourists have weak bonds to their guide and a higher mobility of their own. The tendency to mix with people of the foreign country is higher, we have less segregation.
Global state (gas): At very good knowledge (T) of the foreign country the tourists have no bonds to a guide and do the sightseeing individually, they have a very high mobility of their own. The tendency to mix with people of the foreign country is very high, they are fully integrated.

Economic systems: Companies

Workers: At low mean income (T) employees often are workers in a well ordered and collective group. The rules of work are inflexible. Workers are strongly bound to their boss and make few individual decisions. They do not communicate with many coworkers.
Higher employee: At higher mean income (T) employees work in a group with more individual initiative. The working rules are more flexible. Workers are less bound to their boss and make more individual decisions. Higher employees have to communicate with many workers.
Employer: At very high mean income (T) people are employers (directors) and work only with their own individual initiative. There are no working rules. Employers have to be very flexible and communicate with all workers. The hierarchy corresponds to fig. 1.9. The communication is one directional from the director to the worker.

Political Systems: Countries

Fig. 1.17 shows the standard of living T = GNP/capita in 1995 US$ for all countries as a function of population N (after BARRO and IMARTIN).

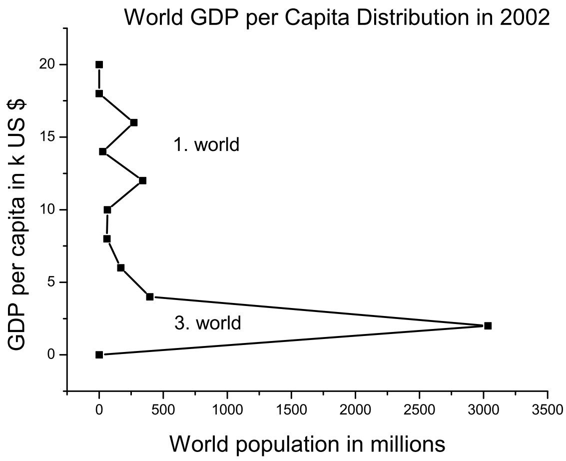

Fig. 1.17 The distribution of the world Gross National Product (GNP) in US$ per person (1995) is quite uneven. N. America, W. Europe, Japan are at the top, democratic and capitalistic. C. Africa, S. E. Asia are very poor and in a hierarchic, non capitalistic state (after BARRO, 1995). The borderline between democratic/capitalistic and non democratic/non capitalistic countries lies somewhere between $2,5$ and 5 kUS$.

1. Countries in North America, Western Europe, Japan and Australia with a high living standard 10.000 US$ per person are capitalistic economies, the people live in a stable democratic state. Democracies are rich, activities are carried out by private individuals and not ordered by the government, the structures of democracy are flexible, a president will stay for one or two elective periods. The mobility of the population is high. Family bonds are weak, grand parents do not generally live under one roof with their grand children, the average number of children is below two, $f < 2$.

2. Countries with a standard of living less than 2.500 US$ (1995) have non capitalistic economies, and people mostly live in non democratic hierarchic structures. Hierarchies are poor, the collective order is high, the structures are generally inflexible, a monarch or dictator is ruling for life time. The mobility of people is low. Families are often large and live all beneath one roof, the fertility is high, $f > 3$.

Obviously, the standard of living acts as an ordering parameter (T) between social states and corresponds to the temperature in materials. At low GNP per capita or standard of living (T) the political state of countries is a collective hierarchy. At high values of (T) states are free, individual democracies.

This result compares to the collective solid state at low temperatures (T) and the individual liquid state at high temperatures. These results lead to an important result:
The standard of living (T) or the gross national product (GNP) per person determines, whether a state may form a stable democracy, or not!

1.3.2
Change of State, Crisis, Revolution

Different states in fig. 1.16 are separated by a phase-transition, a crisis, where the old system breaks down. In religious systems the transition is called a reformation, in political systems the transition is called a revolution. In atomic systems this phase transition is called melting, the ordered solid changes abruptly into the disordered liquid.

1.3.3
Hierarchy, Democracy and Fertility

In fig. 1.18 the standard of living (T) as GNP/person is plotted over fertility (f), the average number of children per woman in a country.

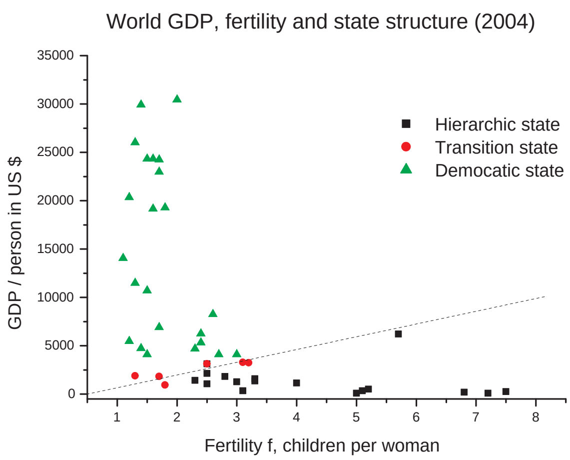

Fig. 1.18 shows the relationship between GNP per person and fertility for 90 states. Countries marked by diamonds are democratic and capitalistic; countries with squares are in a hierarchic state. Countries marked by triangles are in a transition state. The dashed line between democracies and hierarchies represents the line of transition. DSW Weltbevölkerungsbericht (2004).

Fig. 1.18 indicates: there is no common transition point from hierarchy to democracy. But we may draw a straight line between the diamonds of democratic countries and the squares of hierarchic countries. In this way we may divide the world population into two classes:

1. In hierarchic countries with low standard of living the father is head of the family and works outside of the house. Many countries have no pension plans, and only a larger number of children can guarantee a reliable future for parents. The mother stays at home to care for the children as well as for the old parents. The family bonds are strong and very important, like in solids.

2. In democratic countries with high standard of living men and women work hard to obtain the high standard of living. The future relies on their work and the pension plans, not on children. In order to obtain the high living standard people in democratic, capitalistic countries cannot afford to have many children. The mean number of children per family is less than two, $f < 2$. Families also have little time to care personally for their old parents; seniors will live by themselves or in senior homes. Due to the high mobility members of a family often live far apart and family ties are not very close anymore, like in liquids.

The democratic change cannot be forced upon a country, unless the standard of living is raised and the way of life in families has changed. Again we find a close relationship of social and atomic systems.

1.4 Heterogeneous Societies

A society with many different groups is called heterogeneous We will restrict the calculations to binary societies, countries with two ethnic groups like Blacks and Whites in USA, societies with two religions like Catholics - Protestants in Germany or Northern Ireland, markets with buyers and sellers. The calculations may be extended to societies with large numbers of different groups.

1.4.1 The Six States of Binary Societies

In binary systems we have two groups A and B and four different emotional interactions (constraints) between the two groups: e_{AA}, e_{BB}, e_{AB} and e_{BA}. The probability of the combinations $P(A, B)$ is given by Eq.(1.1). N_A is the number of agents in group A, $w_A = N_A/N$ is the probability of being a neighbor of group A; e_{AA} is the interaction AA between members of group A. The same

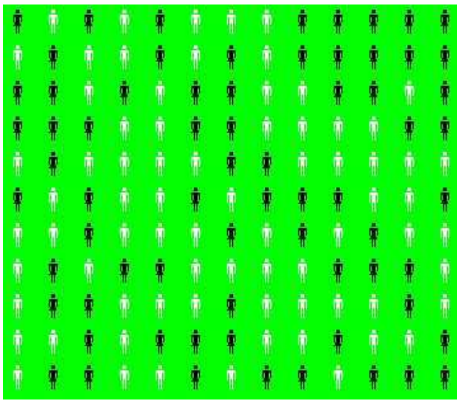

Fig. 1.19 A binary society of women and men or black and white people has four different interactions: AA, AB, BA and BB.

is valid for interactions BB, AB and BA. The Lagrange function (1.2) is given by

$$L(T,N,N_B) = N_A w_A e_{AA} + N_A w_B e_{AB} + N_B w_A e_{BA} + N_B w_B e_{BB} \\ +T\ln\{(N_A+N_B)!/N_A!N_B!\} \rightarrow \text{maximum!} \tag{1.3}$$

$$L(T,N,x) = N\{e_{AA} + x(e_{BB}-e_{AA}) + \epsilon x(1-x) \\ -T[x\ln x + (1-x)\ln(1-x)] + 2\ln 2\} \rightarrow \text{maximum!} \tag{1.4}$$

$$w_B = x = N_B/N; \quad w_A = N_A/N = (1-x) \tag{1.5}$$

$$\epsilon = (e_{AB}+e_{BA}) - (e_{AA}+e_{BB})\,. \tag{1.6}$$

The Lagrange principle (1.6) is the fundamental equation of binary systems that will be applied in all further calculations and simulations of $A\,B$ societies. The Lagrange function L represents the mutual happiness of a binary society about the random distribution of A and B agents. The variable $x = w_B$ stands for the relative size of the minority group (B) and $(1-x)$ the majority group (A). The emotional interactions $e_{AA}, e_{BB}, e_{AB}, e_{BA}$ may be positive or negative. Eq.(1.4) is the Bragg Williams model of regular solutions in physical chemistry and corresponds to the Ising model of magnetic interactions (BECKER 1966) as well as Schelling´s model of social interactions (SCHELLING 1971).

A surprising result of the calculations for binary societies is the parameter ϵ in Eq.(1.6). There are four different interactions, but in binary systems only one parameter, the difference $\epsilon = (e_{AB}+e_{BA}) - (e_{AA}+e_{BB})$ enters the calculations. This parameter ϵ determines the state and the structure of societies.

A detailed discussion of the parameter $\epsilon = (e_{AB}+e_{BA}) - (e_{AA}+e_{BB})$ in Eq.(1.6) leads to at least six different states in binary societies:

1.	Hierarchy:	$\epsilon > 0$	and	$e_{AB} \neq e_{BA}$	
2.	Partnership:	$\epsilon > 0$	and	$e_{AB} = e_{BA}$	$x = 1/2$
3.	Segregation:	$\epsilon < 0$	and	$e_{AB},\ e_{BA} > 0$	
4.	Aggression:	$\epsilon < 0$	and	$e_{AB},\ e_{BA} < 0$	
5.	Integration:	$\epsilon = 0$	and	$e_{AB} + e_{BA} = e_{AA} + e_{BB} > 0$	
6.	Global state:	$\epsilon = 0$	and	$e_{AB} + e_{BA} = e_{AA} + e_{BB} = 0$	

In economic systems (markets) these states correspond to

1.	Hierarchy:	$\epsilon > 0$	and	$e_{AB} \neq e_{BA}$	
2.	Cooperation:	$\epsilon > 0$	and	$e_{AB} = e_{BA}$	$x = 1/2$
3.	Competition:	$\epsilon < 0$	and	$e_{AB},\ e_{BA} > 0$	
4.	Aggression:	$\epsilon < 0$	and	$e_{AB},\ e_{BA} < 0$	
5.	Social market:	$\epsilon = 0$	and	$e_{AB} + e_{BA} = e_{AA} + e_{BB} > 0$	
6.	Capitalism:	$\epsilon = 0$	and	$e_{AB} + e_{BA} = e_{AA} + e_{BB} = 0$	

In binary societies we find again hierarchy, democracy and the global state, like in homogeneous societies. But we also find additional states, partnership, segregation and aggression. These additional states are possible only by mixing different kinds of agents. The model function of human relations in binary societies, Eq.(3.1) depends on six parameters: $L(T, x, e_{AB}, e_{BA}, e_{AA}, e_{BB})$: there are two variables: the standard of living T and relative size of the minority x; and four interaction parameters of sympathy or antipathy between the two groups: $e_{AB}, e_{BA}, e_{AA}, e_{BB}$.
All parameters may change gradually with time. But hey will be regarded nearly constant in the simulations. Only T will be regarded to change with time due to productivity. A change in T again will change the states of the society and lead to transitions of state and to the dynamics of societies.

1.4.2 Partnership

If the attraction to agents of the other group is stronger than to agents of the own group,

$$\epsilon = (e_{AB} + e_{BA}) - (e_{AA} + e_{BB}) > 0\,. \tag{1.7}$$

Fig. 1.20 shows the $AB\,AB$ structure of a folk dance group of boys and girls. In fig. 1.21 the function $L(x)$ – Eq.(1.4), represents the happiness of a group of boys and girls at a square dance. The relative number of girls is x and the relative number of boys is $(1 - x)$. If no girls show up, the happiness $L(x = 0)$ of the group will be low, if only girls attend the happiness $L(x = 1)$ will also be low. The maximal happiness of the group will be obtained, if the number of boys and girls are equal, $L(x = 1/2)$, when everyone has a partner.
Partnership looks very much like hierarchy, in fact there is no way to detect the difference between partnership and hierarchy from the outside. A married

Fig. 1.20 A folk dance group of boys and girls show the AB AB structure of partnership. Girls are more attracted to boys and boys to girls.

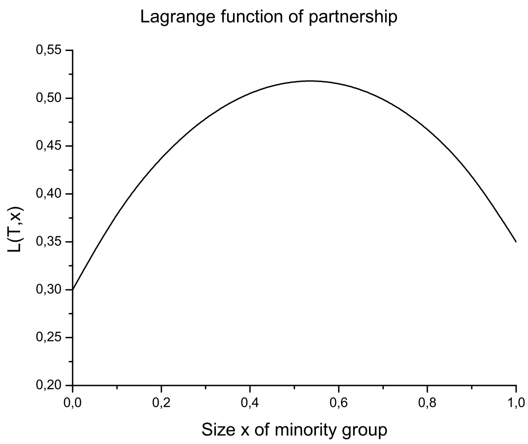

Fig. 1.21 The Lagrange function $L(x)$ – Eq.(1.4), corresponds to the happiness of girls and boys at a square dance, x is the relative number of girls. If no girls show up, the happiness $L(x = 0)$ of the boys is low. If there are only girls, the happiness $L(x = 1)$ is also low. The maximum happiness is obtained for equal numbers of girls and boys at $L(x = 1/2)$, when everyone has a partner.

couple may live in true partnership or in (male) hierarchy, the checker board structure of the systems are alike. A couple may be dancing in the crowd of dancers, but we do not know whether they are at equal level or whether he leads her or vice versa. The difference between partnership and hierarchy can only be detected by additional inside information.

1.4.3
Integration

If the attraction to the other group is the same as for the own group,

$$\epsilon = (e_{AB} + e_{BA}) - (e_{AA} + e_{BB}) = 0\,. \tag{1.8}$$

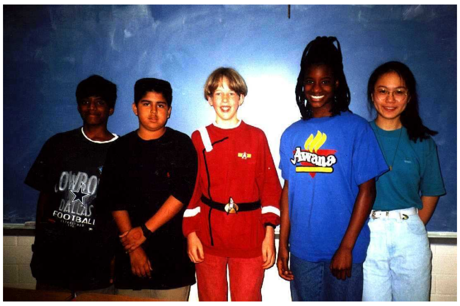

Fig. 1.22 An integrated class of young people from all over the world. The kids are friends and the happiness of each person in the class is independent of the kind of neighbor.

Fig. 1.22 shows an integrated class of young people.

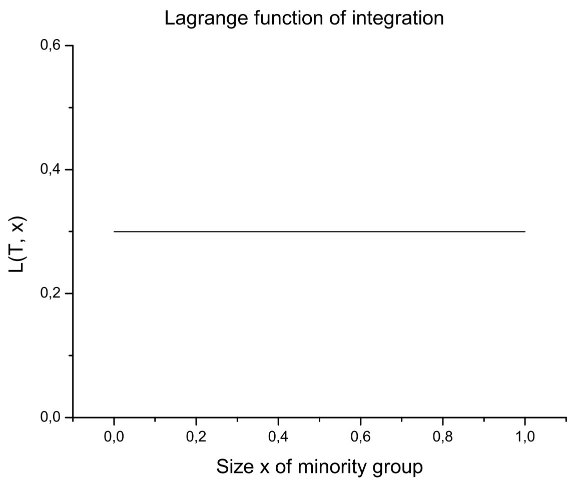

Fig. 1.23 For $\epsilon = 0$ the Lagrange function $L(x)$, Eq.(1.4), corresponds to the happiness of a group of girls and boys in front of the refreshments as a function of the relative number x of girls. The happiness L in the refreshment lines is constant and independent of the number of boys or girls in the lines.

The Lagrange function in fig. 1.23 represents to the happiness of a group of boys and girls in the lines for soft drinks after dancing. For thirsty girls or boys in the lines it does not matter, who is in front or behind, it only matters to get a soft drink, soon. The happiness of thirsty dancers is constant, fig. 1.23, independent of the number of girls in the line,

1.4.4 Segregation

If the attraction to the own group is stronger than to the other group,

$$\epsilon = (e_{AB} + e_{BA}) - (e_{AA} + e_{BB}) < 0\,. \tag{1.9}$$

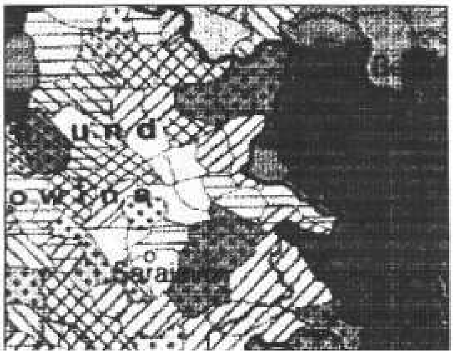

Fig. 1.24 Map of Bosnia 1991. The Bosnian society is segregated into mainly Moslem population (bright areas) and mainly not Moslem population (dark areas). (With kind permission, Westermann Schulbuch Verlag, Braunschweig.)

Bosnia in fig. 1.24 is a mixture of Moslems and Christian people. Brass in fig. 1.25 is a mixture of copper and zinc atoms. The structural similarities in both figures is obvious and corresponds to segregation in binary systems. The simulation for $\epsilon < 0$ is given in figs. 1.26 and 1.27.

Fig. 1.25 Surface of a brass probe after etching. The brass alloy is segregated into areas with mainly zinc (bright regions) and areas with mainly copper (dark regions).

The Lagrange function or happiness in fig. 1.26 has two maxima, one for the minority of Moslems in Christian areas, the other for the Christian minority in the Moslem areas. For nearly equal number of Moslem and Christian people, $x = 1/2$, the maximum of the segregated system is given by the diamond on the common tangent. This diamond value is higher than the function $L(x = 1/2)$ for random distribution of agents, as shown by the solid line. Fig. 1.27 ist obtained by Monte Carlo simulation of the Lagrange function (1.4).

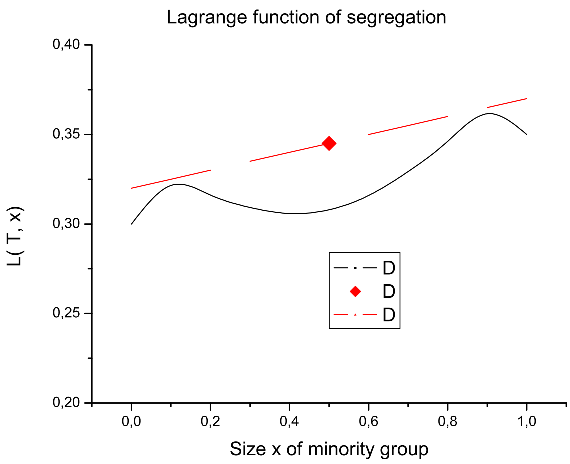

Fig. 1.26 Lagrange function (1.4) of a binary society with preference for the own groups, $\epsilon < 0$. The function shows two maxima, for the minorities in each section.

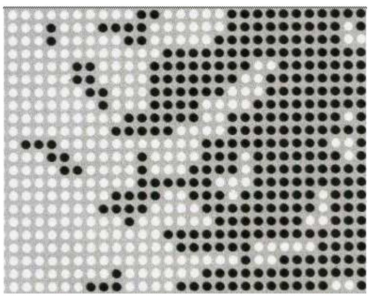

Fig. 1.27 Monte Carlo simulation of the Lagrange function $L(x)$ of the distribution of Moslems and Christians in Bosnia in fig. 1.23.

1.5 Dynamics of Societies

1.5.1 Hierarchy and Opinion Formation

In elections (opinion growth) the winning party leader determines the political direction of more than 100 million people, fig. 1.28.

Fig. 1.28 Elections are a good example of opinion formation. The winning leader determines political direction of more than 100 million people.

Fig. 1.29 A rock crystals is a well known example for crystal growth. The winning nucleus determines the crystal direction of more than 10^{23} molecules.

This corresponds to the crystallization of liquids. In crystal growth one nucleus determines the crystal direction of 10^{23} molecules, fig. 1.29. Crystal growth and opinion formation may both be modelled from the Lagrange function (1.2), $L = E + T \ln P \rightarrow$ maximum:

$$L(T, x_i, x_k) = N\{\Sigma e_{ik} x_i x_k - T\,\Sigma x_i \ln x_i\} \rightarrow \text{maximum!} \tag{1.10}$$

$$e_{ik}(\circ\bullet) = e_{ik}(\bullet\circ) > 0 \text{ (attractive)} \tag{1.11}$$

$$e_{ik}(\bullet\bullet) = e_{ik}(\circ\circ) < 0 \text{ (repulsive)} \tag{1.12}$$

An ideal checker board structure can start with $\circ\bullet$ = Bush or with $\bullet\circ$ = Kerry.

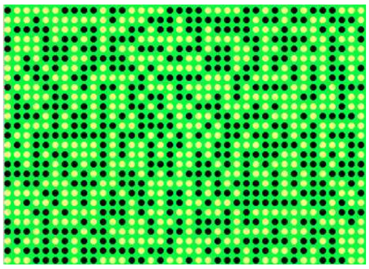

Fig. 1.30 Chaotic distribution of light and dark balls. The distribution corresponds to a disordered binary liquid of GaAs or an undecided crowd before the elctions (yes-no; Bush-Kerry): ○● = Bush; ●○ = Kerry; ○○ = ●● = undecided.

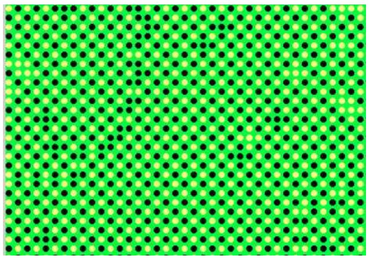

Fig. 1.31 Disordered checker board distribution of light and dark balls. The pro Kerry opinion ○● – see upper left corner – and the pro Bush opinion ●○ in the center are separated by a curved line of undecided. Both areas contain also dissidents = ● ○ ○ ○ ● or ○ ● ● ● ○. The pattern simulates the 2004 US elections or crystal growth of a gallium arsenide with grain boundaries and antisites.

Fig. 1.30 corresponds to the situation before the election, there is no checker board structure, no preference for Bush or Kerry, most people are still undecided. Figs. 1.31 and 1.32 are created by a Monte Carlo method. A random agent is asked, if he wants to switch positions with his left (right, lower, upper) neighbor. If the Lagrange function Eq.(1.10) is raised by this local movement, the neighbors will switch. The parameter T and the concentrations x_i ($\circ$) and x_k ($\bullet$) remain constant. After some time the pattern of fig. 1.31 will emerge. The local preference for Bush or Kerry will grow, until both areas meet and are separated by undecided agents. The lines of opposite opinions are not always stable. In a curved line there are more agents with one opinion than inside with the opposite opinion. With enough time the curved line is pushed out and only one opinion will dominate. A straight line with equal numbers of opponents, fig. 1.32, will be stable for long times and produce a stable two party system.

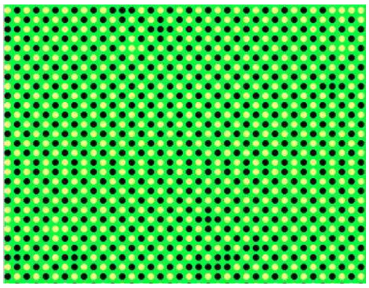

Fig. 1.32 A stable twin formation ○● (see upper corner) and ●○ (lower part). The border line is straight. Both opinions, parties, armies contain local antisites or dissidents. The straight line has equal numbers of opponents and leads to a twin crystal, a stable two party system or an undecided battle front.

1.5.2
Simulation of Segregation

Phase Diagrams

Phase transitions in homogeneous materials are called melting or vaporization. These are often first order transitions, which are accompanied by a heat of transition and a sudden change of order.
Phase transitions may discussed on the basis of the Lagrange principle, Eq.(1.4). At equilibrium or maximum the first derivative of L with respect to x will be equal to zero, if $e_{BB} = e_{AA}$,

$$\epsilon(1-2x) - T[\ln x - \ln(1-x)] = 0 \tag{1.13}$$

Eq.(1.13) may be solved for T

$$T = +\frac{\epsilon(1-2x)}{\ln x - \ln(1-x)} \tag{1.14}$$

Eq.(1.14) has been plotted in the phase diagram, fig. 1.33. The plot shows the temperature T that is needed to dissolve a quantity x of platinum in gold.
Mixing gold and platinum is very much like mixing tea and sugar. In cold tea only a little bit of sugar may be dissolved, the rest will segregate at the bottom of the cup. In hot tea much more sugar may be dissolved.

Intermarriage

Intermarriage data are the phase diagrams in social systems. The diagrams tell us about the degree of integration in a society, they are derived from:

$$P(x) = 2x(1-x) \tag{1.15}$$

Equation (1.15) of intermarriage is a parabola and corresponds to Mendel´s law of mixing white and red peas. Fig.1.34 shows the parabola of intermar-

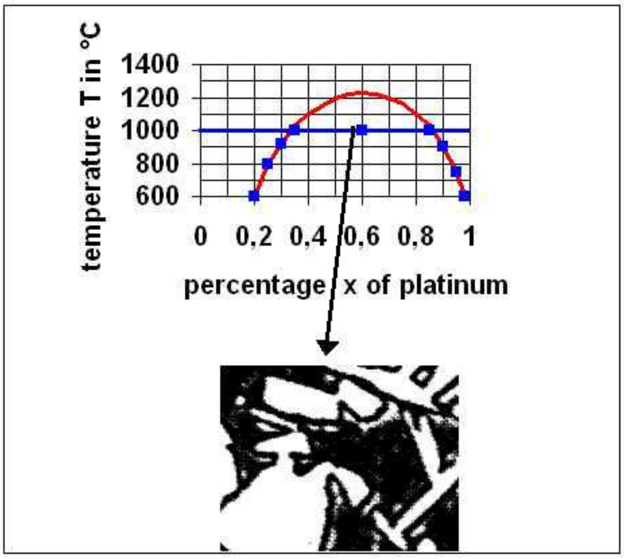

Fig. 1.33 Phase diagram of gold platinum alloys (HANSEN, 1958): The curve $T(x)$ corresponds to the temperature that is needed to dissolve the percentage x of platinum. At $600°C$ gold may solve only 20% Platinum. At $1000°C$ the "solubility limit" is $x = 0,35\%$ or 35$ platinum. Any higher percentage will lead to segregation, as shown by the bright areas with mainly gold and dark parts with mainly platinum.

riage for Catholics and Non Catholics in 10 different states of Germany and in Switzerland in 1991. Data points on the parabola correspond to states, where Catholics and Non Catholics are integrated. This is observed for states below $x = 0,2$ or 20% Catholics. The value $x = 0,2$ represents a "integration limit" of Catholics in the German society. The constant rate of intermarriage at 33% corresponds to the equilibrium temperature in fig. 1.33 and indicates the equilibrium in the mutual religious tolerance T of different neighbors. In states with a higher percentage of Catholics we find segregation into mainly Catholic and mainly Non Catholic areas, as shown on the map of the state of Westphalia with 40% Catholics. White areas have $x = 0,2$ or 20% Catholics and about 80% Non Catholics and dark areas 80% Catholics and 20% Non Catholics. So everybody is most happy to live in the right neighborhood (MIMKES, 2000). The intersections of the parabola at $x = 0,2$ and $x = 0,4$ corresponds to the maxima in the Lagrange function, fig. 1.25.
Mixing tea and sugar or platinum and gold in fig. 1.33 corresponds to the mixing Catholics and Non Catholics, fig. 1.34. The maximum rate of intermarriage at the "integration limit" x is equivalent to the solubility limit of sugar in tea or platinum in gold, fig. 1.33.

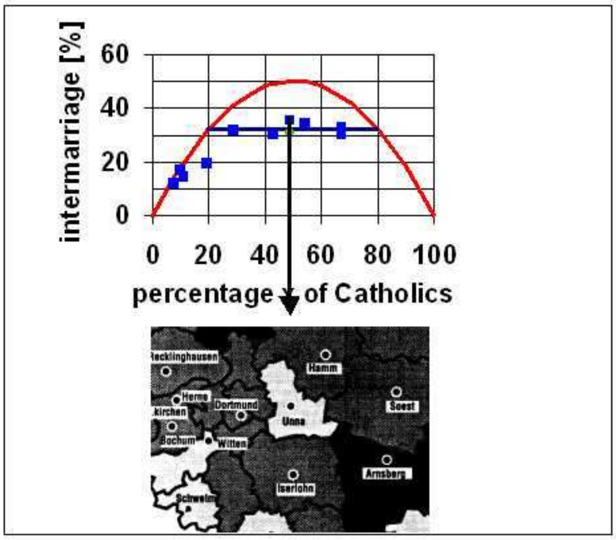

Fig. 1.34 Intermarriage between Catholics and non Catholics in 10 different states of Germany in 1991 and in Switzerland 1992. Intermarriage is ideal up to the "integration limit" of $x = 0,2\%$ or 20% Catholics in any state of Germany. For states with higher percentage we find segregation. In for Westphalia with 40% Catholics. Westphalia segregates into black areas with mainly Catholics and white areas with mainly Non Catholics (Germany, 1991).

Eq.(1.15) is equivalent to SHELLING´s model (1971). However, the present model does not only give a detailed explanation for Schelling´s constant c, but also leads to solutions of segregation problems.

The intermarriage diagrams of African and Non African Americans in 33 different states of the US (1988) is given in fig. 1.35. Again we find an "equilibrium temperature" for mixing African and non African Americans at a portion $P = 1,1\%$ of intermarriage. The value of $P_{\max}$ is independent of the number of African Americans in the particular states and proves again the existence of the Lagrange parameter or mutual racial tolerance T.

1.5.3
Simulation of Aggression

Fig. 1.36 shows the intermarriage diagram between 40% Catholics and 60% Non Catholics in Switzerland, Germany and Northern Ireland in 1991. The mutual rates of tolerance are 32% in Switzerland and Germany and 2,3% in Northern Ireland! This low value indicates strong negative emotions and the danger of aggression. In all highly segregated societies with low tolerance like Ireland, Israel or Bosnia civil war can erupt at any time.

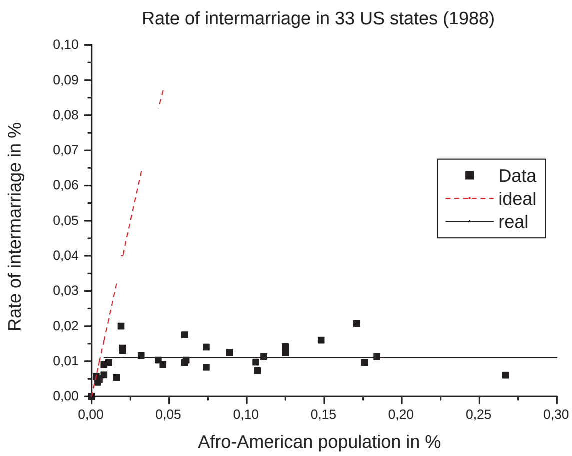

Fig. 1.35 Intermarriage between African and Non African Americans in 33 States of the US in 1988. Intermarriage is ideal up to the "solubility limit" of $x = 0,55\%$ African Americans in each state. For states with higher percentage we find a constant portion of intermarriage, $I = 1,1\%$, indicating the "social equilibrium temperature" between different states of the US. The "solubility limit" leads again to segregation into predominant white and predominant African American areas. (U. S. Bureau of the Census 1990).

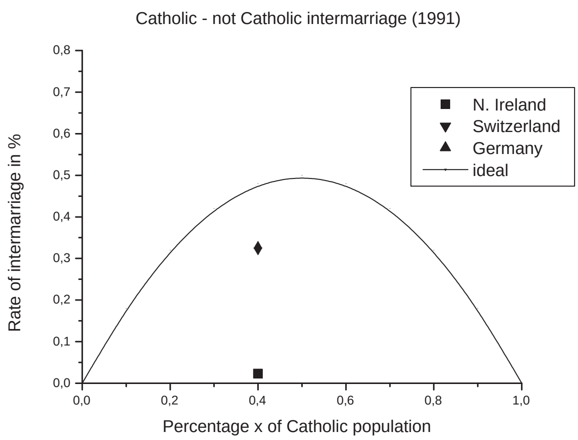

Fig. 1.36 The three countries Germany, Switzerland and N. Ireland have 1991 the same percentage of 40% Catholics. But the rate of intermarriage between Catholics and Protestants in Northern Ireland is $I = 2,3\%$, compared to $I = 32\%$ in Germany and Switzerland and reflects the low tolerance, danger of aggression and civil war in each country. (Germany, 1991; Switzerland, 1992)

The process of segregation may be simulated by the Lagrange equation Eq.(1.4). This way we can observe, how the process of segregation is reflected by the

decreasing rate of intermarriage. Starting from a completely integrated minority intermarriage and tolerance drops from the maximum to a minimum value due to segregation or aggression. This special simulation gives a proper interpretation to the danger of aggression and civil war due to changing emotions within the population.
If emotions to other groups become negative, like for hate, envy, distrust or aggression, we have the condition for aggression,

$$\epsilon > 0 \text{ and } (e_{AB} + e_{BA}) < 0 < (e_{AA} + e_{BB}) \tag{1.16}$$

Negative emotions to others will lead to total separation of the groups, and vice versa, a total separation of groups will generally lead to aggression. This is presently observed in many places with binary populations like Bosnia, Northern Ireland or Israel.
The close relation between separation and negative emotions is also observed in the popular prisoner dilemma of game theory (AXELROD 1984). The separation of prisoners generally leads to distrust, $\epsilon < 0$ and defection.

1.6 Conclusion

In the thermodynamic formulation of social science the laws of societies have been derived from statistical laws under constraints. No further assumptions have been made. The calculated functions are supported by data, which all seem to agree very well. The results indicate that natural science may very well be applied to social science. The thermodynamic approach, which has been most successful in all natural sciences like physics, chemistry, metallurgy, meteorology, biology and engineering is also a successful basis in the fields of socio-economic sciences. The approach to social science is very general and the data seem to fit well. In this view the Lagrange statistics seems to be a valuable addition to the standard statistical theory of social science.

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2
Figures and Tables

Author list

Figures and tables are floating objects. Their position on the page is controlled by an optional parameter list of the form [htbp]. One-column captions (for four lines or less) are created by `\Caption{...}`, two-column captions by `\Captiontwo`.

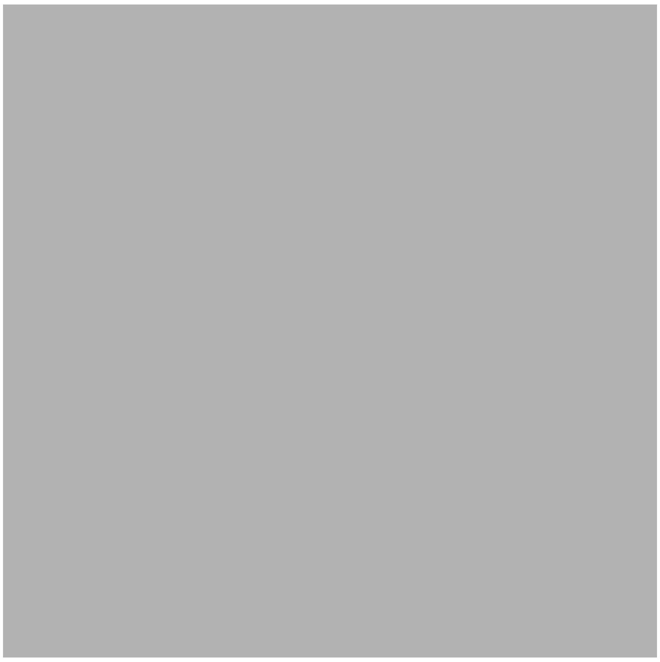

Fig. 2.1 Figure caption are set automatically to a width of 75% of the page width. This value can be increased to 100% in case of very large figures or very large figure captions (see the following figure).

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Fig. 2.2 Figure caption are set automatically to a width of 75% of the page width. This value can be increased to 100% in case of very large figures or very large figure captions. Figure caption are set automatically to a width of 75% of the page width. This value can be increased to 100% in case of very large figures or very large figure captions. Figure caption are set automatically to a width of 75% of the page width. This value can be increased to 100% in case of very large figures or very large figure captions. Figure caption are set automatically to a width of 75% of the page width. This value can be increased to 100% in case of very large figures or very large figure captions. Figure caption are set automatically to a width of 75% of the page width. This value can be increased to 100% in case of very large figures or very large figure captions. Figure caption are set automatically to a width of 75% of the page width. This value can be increased to 100% in case of very large figures or very large figure captions. Figure caption are set automatically to a width of 75% of the page width. This value can be increased to 100% in case of very large figures or very large figure captions. Figure caption are set automatically to a width of 75% of the page width. This value can be increased to 100% in case of very large figures or very large figure captions. Figure caption are set automatically to a width of 75% of the page width. This value can be increased to 100% in case of very large figures or very large figure captions.

The latex code for two-column captions is:

```
\begin{figure}[t]
\def\capfrac{1}% <----- Sets the caption width to 100%
\includegraphics[width=.75\textwidth]{dummy.eps}
\Captiontwo{
Figure caption are set automatically to a width of 75\%
of the page width. This value can be increased to 100\%
in case of very large figures or very large figure
captions.
[...]
}
\end{figure}
```

The one-column figure caption is created by entering "`\Caption`" and the two-column figure caption is created by entering "`\Captiontwo`" inside the figure environment, both with a capital C. The command `\def\capfrac{1}` sets the width of the caption to 100% of the page width.

The default format for figures is the eps format.
Please check whether the figures are correctly cropped, i.e., that the figure is not surrounded by additional white borders.

Table 2.1 Tables as floating objects

Table head			
a	b	A	B
c	d	C	D

The LaTeX source for the table is:

```
\begin{table}[t]
\caption{Tables as floating objects}
\begin{small}\sffamily
\begin{tabularx}{4.4cm}
{@{\extracolsep{\fill}}llll@{}}
\hline
\noalign{\vspace*{1mm}}
\multicolumn{4}{c}{\textbf {Table head}}\\
\noalign{\vspace*{1mm}}
\hline
\noalign{\vspace*{1mm}}
a & b &A &B\\
c & d &C &D\\
\noalign{\vspace*{1mm}}
\hline
\end{tabularx}
\end{small}
\end{table}
```

3
Endnotes, Bibliography and Subject Index

Endnotes and subject index are set as unnumbered chapters in two-column mode starting on a right page.
The bibliography can appear in two forms. In multi-author books, the bibliography, here called References, is in general the last section of a chapter with the contribution of one or more authors. References do not have to start on a new page. In both single- and multi-author books, a bibliography referring to the whole book can also appear as a chapter at the end of the book.
The name can be adapted by the following command:

```
\renewcommand{\bibname}{References}
```

or

```
\renewcommand{\bibname}{Bibliography}
```

References on a section level at the end of a chapter are set by entering

```
\begin{thebibliography}

\end{thebibliography}
```

and a bibliography on a chapter level as an own chapter

```
\begin{Thebibliography}

\end{Thebibliography}
```

Empty pages at the end of a chapter can be created by

```
\cleardoubleemptypage
```

or

```
\newpage
\thispagestyle{empty}
~
\newpage
\thispagestyle{empty}
```

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3.1

This is an even page. It should be empty.

Notes

1) endnote example

2) unnumbered equation

This is the place for the endnotes. The source code for this chapter is:

```
\renewcommand{\notesname}{Notes}

\theendnotes %  <--- This command opens a new chapter Notes

\chaptermark{\notesname}%
\sectionmark{\notesname}%
```

It follows a section with references. A bibliography on a chapter level follows on page 46.

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The LaTeX source for the bibliography is:

```
\def\bibindent{6mm} % room for up to 3 digits

\begin{thebibliography}
\bibitem{xxx}
[...]
\end{thebibliography}
```

3.2 The Subject Index

Terms are marked as usual with the `\index` command. In order to create the *.ind file with the sorted index enter the following command on the command line:

```
makeindex -s format170x240.mst <base_filename>
```

The LaTeX source code for the the index chapter is:

```
{
  \addtocontents{toc}{%
    \protect\tocline
      {2}%
      {}%
      {\indexname}%
      {\arabic{page}}}%
\footnotesize
\printindex}
```

4
Chapter Heading and Author List for Multi-Author Books

First Author, Second Author, and Third Author

The input for the chapter heading and the author list is:

```
\chapter{Chapter heading and [...] multi-author books}
\chapterauthor[F. Autor, S. Author, and T. Author]
{First Author, Second Author, and Third Author}
```

The part in brackets of the `\chapterauthor` command is optional and contains the names of the authors as they will appear in the table of contents under the respective chapter title. If the optional part is empty, i.e., the command has the form `\chapterauthor[]{authorlist}`, no authorlist will be set in the table of contents. If the optional part is missing, i.e., the command has the form `\chapterauthor{authorlist}`, the authorlist will be identical both in the table of contents and below the chapter heading.
Footnotes consisting of up to six lines are created as usual with the `\footnote` command and set in a one column mode spanning 75% of the page width (see below).
Here comes a footnote.[1] Footnotes are set at the bottom of the page. Endnotes, however, are collected at the end of the book in an own section before the bibliography. This is a endnote[1]. (see the chapter Notes on page 39).
Footnotes consisting of more than six lines[2] should be set in a two-column mode. The placement and numbering of two-column footnotes is accomplished by entering the commands
`\pagefootnotes{ \setcounter{footnote}{1} \footnotetext{...}}`.
It is also possible to collect several two-column footnotes and write them out by entering the following commands
`\pagefootnotes{\footnotetext{...}\footnotetext{...}}`.

1) There is a footnote and a endnote counter. Both can be used in the same document and they are numbered differently. Footnotes span 75% of the page width.

2) There is a footnote and A endnote counter. Both can be used in the same document and they are numbered differently. Footnotes span 75% of the page width. Footnotes are set at the bottom of the page. Endnotes, however, are collected at the end of the book in an own section before the bibliography: (see the chapter Notes on page 39).

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Unnumbered chapter heading

4.1
Section Heading

Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text.

4.1.1
Subsection Heading

Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text.

4.1.1.1 Subsubsection Heading

Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text. Some text.

A Paragraph This is a paragraph.

A subparagraph This is a subparagraph.
Formulas are centered

$$\operatorname{div} \vec{E}(\vec{r}, t) = 0 . \tag{4.1}$$

This results in[2]

$$\operatorname{rot} \frac{\partial}{\partial t} \vec{E}(\vec{r}, t) + \frac{\partial^2}{\partial t^2} \vec{B}(\vec{r}, t) = 0 .$$

The first page of a chapter contains a copyright line at the bottom of the page. The data of the copyright line (ISBN number, name of the author(s), title, copyright year) are provided by the publisher. Please enter the data into the definition in the preamble of your master file.

```
\def\copyrighttext{\textit{Titel}. Author(s)...}
```

In multi-author works, a chapter is usually closed by a bibliography. For a usual bibliography at the end of the book see page 46.

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