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ARTICLE

An Evolutionary Theory of Economic Interaction —Introduction to Socio- and Econo-Physics

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Abstract

We try to construct an evolutionary theory of economic and social interaction of heterogeneous agents. Modern physics is helpful for such an attempt, as the recent flourishing of econophysics exemplifies. In this article, we are interested in a specific or more fundamental use of physics rather than in the recent researches of econo-physics. In the first part of this article, we mainly focus on the traditional von Neumann-Sraffa model of production as complex adaptive system and examine the measure of complexity on this model in view of thermodynamical ideas. We then suggest the idea of hierarchical inclusion on this model to define complexity of production. In the latter part, we try to construct an elementary theory of social interaction of heterogeneous agents in view of statistical mechanics.

Keywords: heterogeneous interaction, complex adaptive system, thermodynamics, complexity of production, truncation of a production system, hierarchical inclusion, social temperature.

1. The Scenarios on Evolution and Selection

1.1 The stories of evolution

Both society and economy can evolve. When he argued about what Evolutionary Economics was, Hodgson (2003) suggested that there was *a priori* a set of possible variations for the selection of species in Darwinism, what we call an *ex ante* variation. Following Hodgson, here, we should not demonstrate unilaterally that a certain variation could emerge *ex post* as a result of the selection process on species or social subgroups. Put another way, a newly arriving variation will emerge as the result of interacting variations. Hodgson thus thinks that there unambiguously exists a certain demarcation between Darwinism and Schumpeterian Economics, because Schumpeterian economics gives importance rather to resultant variation.

Sometimes we have the idea that an agent which happened to survive may be proven

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to be *ex post* superior. Even given superficially homogeneous agents, indeed, we could verify the emergence of *mutants*. This is just a Spencerian story of individualistic agents, as Keynes (1972, p. 28) heavily satirized. One that can perform better by itself than others can survive to be a *mutant*. This idea may then be reinforced in association with the utility maximizing principle of individualistic homogeneous agents. The importance of marginal productivity in this context is just an equivalent to the story of the *giraffe*: The giraffe, with its higher neck, has the advantage of being able to eat more leaves, and can thus survive more easily. Without heterogeneity, thus, we are sometimes used to seeing the occurrence of *mutants*.

1.2 The internal process of selection: ex ante variations, diversities, and niches of species or social subgroups

Suppose on the other hand that there were *ex ante* many heterogeneous agents. In this case, we have a new utility theory of interaction coming from a multinomial logit model, even if we leave the domain of utilitarianisms.[1] The idea of heterogeneous agents is not necessarily inconsistent with utilitarianism. Heterogeneous interacting agents lead to a new set of variations, anyway.

We thus have **heterogeneous interacting agents** sometimes with bounded rationality, while, **homogeneous individualistic agents** always with complete rationality.[2]

In the process of selection facing diversity, we will need to comprehend an internal process to delete a niche and add to a new agent, to more or less restore the missing interaction. Holland, who suggested the framework of the *complex adaptive system*, described it as follows:

The diversity is neither accidental nor random. The persistence of any individual agent, whether organism, neuron, or firm, depends on the context provided by the other agents. Roughly, each kind of agent fills a niche that is defined by the interactions centering on that agent. If we remove one kind of agent from the system, creating a "hole," the system typically responds with a cascade of adaptations resulting in a new agent that "fills the hole." The new agent typically occupies the same niche as the deleted agent and provides most of the missing interactions (Holland, 1995, p. 27).

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[1] The multinomial logit model can be compatible with the sequential choice model, as the Luce model (Luce, 1959) suggested. The multinomial logit model has a comprehensively generic feature either to be derived from the random utility model or the Luce model. See Bierlaire (1997) for details.

[2] Bounded rationality may be given in view of the failure of estimation on the secondary effects. If we had some subpopulation which failed to catch up with the secondary effects of the interaction, the opponent subpopulation should rather utilize the failure of the former to its advantage.

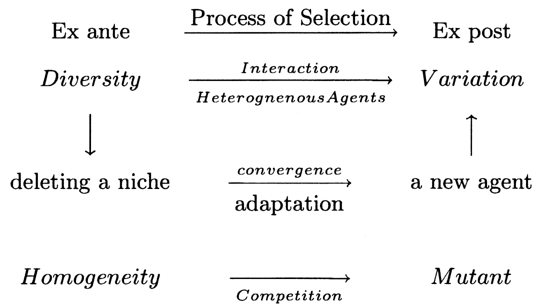


Fig. 1. A scheme of selection and variations.

The visual formula of our main points may be summarized in Fig. 1:

1.3 Rules choice

Principle in the above may be interchangeable with *rule*. The maximizing principle can be read as a maximizing rule, for instance. Rule choice in economics from the first has been predetermined for the last decades. There then is no other room than the utility maximization rule in orthodox economics. If we should accept this, our research focus should be enclosed within the limited world of Descartes, where realism of material substance and spiritual agent are completely separated. *Empiricism* thus was inclined to be almost entirely removed from economic theory. Games of purely mathematical modeling then continued to be played long.

We know that aggregation of microscopic agents by itself implies the emergence of a new property which never appears in the state of a single agent. This is a true meaning of aggregation. We can, however, notice that our attitude to study the Macroscopic Aggregate Production Function was quite different from this. Hoover (2001, pp. 74–85) discussed such an attitude in details. We can also wonder to ourselves:

Why did economists have much fun arguing the same macroscopic properties as the microscopic properties of production? This is an inexplicable question in the traditional economics. In the end of the last century, however, this trend just began to be checked by a new stream from Physics. This stream symbolically means the rise of *econophysics*.

In orthodox economics, we are only permitted to use the sole rule, i.e., *utility* maximization as the *generic* principle. Utility must, however, be a partial principle.

We must stop applying a straitjacket, because we will require another principle such as *risk* minimization when we are faced with uncertainty. Diverse rule choice should be allowed.[3)]

1.4 Historical excurses for econophysics

At the end of the last century, the Santa Fe Institute was well-known worldwide for holding a workshop in September 1987 titled: "Evolutionary Paths of the Global Economy" generated by P. W. Anderson, K. J. Arrow, and David Pines (1988). This conference was a historically monumental meeting to bring together economists and physicists. The success of this meeting has spurred research on the economy as an evolving complex system.

Independently of the Santa Fe attempts, however, in Stuttgart, Germany, it must be noticed that we had a big bang of the new approach to social phenomena. This approach was born in the process of the Synergetics Project, stimulated, in particular, by Herman Haken's study on the laser beam. This group calls its own approach Sociodynamics. Wolfgang Weidlich has played the decisive role in **the Sociodynamics Project**, which was compiled in Weidlich (2000).

Interestingly, this nomination coincides with the final layer of Schumpeterian economics, since we have the triangular theoretical layers of statics, dynamics, and sociodynamics as the stages of economic epistemology. Evolutionary economics may thus easily accept the study of sociodynamics, while the synergetics approach may accept evolutionary economics if it becomes engaged in any integration of physics and economics.

We can normally distinguish the variables of a dynamical system between *slow* variables and *fast* variables. By referring to slow variables, the Synergetics approach in brief is used to find **the order parameter** in order to construct **the master equation** for studying the dynamical properties of a system. The sociodynamics approach thus introduced the idea of a master equation into social and economic analysis. In other words, the idea of statistical mechanics could, together with the master equation, be applied to social science. This approach never depends on so-called classical mechanics. In the new approach, each state could be described in **a detailed balance** of in-flow and out-flow, and the emergence of **fluctuations** of state can also be analyzed.

[3)] *The multi-armed bandit problem* (see Bellman (1961)), as Holland (1992, Chapter 5) referred to is a good example that optimization should fail. Our permissible option in the environment that the problem is faced with is limited to risk minimization.

2. Entropy and Complexity of the Economic System

2.1 Diverse rule choice of an economic system

An example of diverse rule choice may be cited from the *thermodynamical* view. In the earlier era of neoclassical economics, however, they had the idea of an *ex ante* production function as well as *ex post*. Here we can take the idea of **complexity** as **a rule**. In other words, we introduce complexity as **a new coordinate**. This process will lead us to define a new approach to Evolutionary Economics. We start from the point.[4)]

Now we can define **the entropy of mixing types** as follows:
N different elements: $N_1, N_2, \cdots, N_n$.
M different classes (or types): $M_1, M_2, \cdots, M_m$.
Entropy S then is defined by the use of the probability P of the distribution of the N elements in M classes of categories: $S = N \ln N - N_i \ln N_i$

In the cyclic process of economic production $\{A \to B \to A\}$, the first part $A \to B$ indicates *productive arrangements* while the second part $B \to A$ does the process of *expenditure*.

$$-\oint \Delta W = \oint \delta q = \int_A^B \delta q_{(1)} + \int_B^A \delta q_{(2)} = \int_A^B \delta q_{(1)} - \int_A^B \delta q_{(2)} = \Delta q$$

In **the first law of thermodynamics**, economic net output Δq is only possible by work or production. But it is impossible to calculate *ex ante* how much work we will gain, because the output depends on the production process and there are so many possibilities for this process. *Ex post* we may calculate the net output Δq, as we then know which process has been carried out. **The second law** tells us about the integrating factor T (temperature) that will transform a not exact differential form into an exact form $dS = \delta q / T$. This means the function S is independent of the path and may be calculated *ex ante*.[5)]

Under these macroscopic laws, we have different cycle paths, as shown in Fig. 2. We shall mainly investigate the way a path could appear by means of inspecting each *inner mechanism* accompanying each different path. That is to say, we focus on the link between the selection of productive activities and the integration of subsystems (*hierarchical inclusion* as defined later).

2.2 Truncation and macroscopic orders

In the event, we will then have the insight that:
(a) The way to take a Carnot path of production may depend on the size of Δq as well as

[4)] This section is a concise version of Aruka and Mimkes (2005).

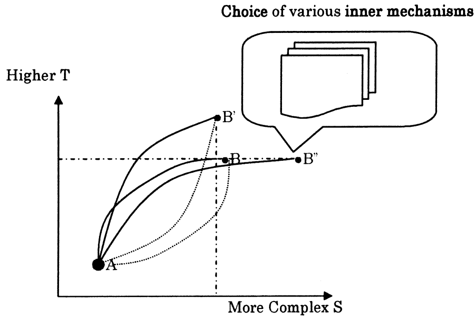


Fig. 2. Different cycle paths with different inner mechanisms.

T. It is important for Δq which coordinate we could cut the cycle $\{A \rightarrow B \rightarrow A\}$ at, which value of B we could choose.

(b) **The truncation of production system** implies a kind of admissible number of combinations of production activities (processes). The feasibility of *truncation* decisively depends on the rate of profit (the rate of surplus) and the price system chosen under a given rate of profit. The precise analysis was smartly proved by Schefold (1989). As we soon see in Aruka and Mimkes (2005), truncation of production system in association with the idea of **hierarchical inclusion** can lead to **complexity of production**.

(c) Hence we have $(a) \Leftrightarrow (b)$.

2.3 Hierarchical inclusion of productive subsystems

In our attempt, we try to replace this idea of entropy of production with a more concrete idea of **complexity** from the economic point of view. In particular, we use the idea of **hierarchical inclusion**. This can be summarized in Table 1.

We can characterize four different states by a couple of factors, i.e., low and high, on the coordinates of **entropy** and **temperature**. We find there both a state associated with a high temperature but low entropy and another state with a high temperature. This kind of characterization may easily be applied on the plane of **complexity** and **average welfare**, as shown later in Fig. 3. Thus we notice the case of a simpler production with higher profitability. Hence, higher profitability does not necessarily require a more complex production. Observe an economic system of a higher GDP combined with a simpler

[5] S represents the different possibilities for production activities (processes). And we later find $S = \ln P$. In a finite system of N elements the number of possibilities may be calculated by the law of combinations $P = N!/N$.

Table 1. Complexity and Profitability

	Complexity	
Average welfare	Simple	Complex
lower	$\{\alpha_1\}^1$	$\{\alpha_1, \alpha_2\}^2$
higher	*	$\{\alpha_1\}^1 \cup \{\alpha_1, \alpha_2\}^2$ hierarchical inclusion

order as indicating a country producing only crude oil. There may, on the other hand, also be a system of a lower GDP with a more complex system, which implies a system of production by means of *many* commodities. In a more integrated view of production, thus, we can regard each system as a *subsystem*. In this case, we may have an *ensemble* of economically *revealed* subsystems.

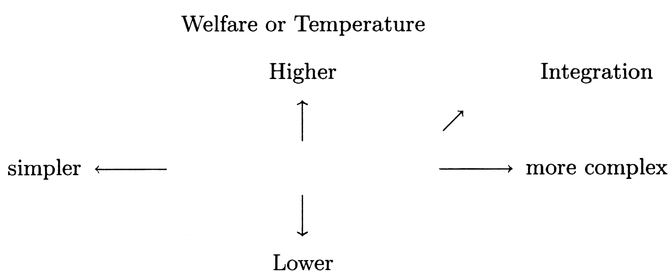


Fig. 3. Introduction of complexity and temperature as the new coordinates.

If we are facing an optimization problem like maximization of the net domestic income subject to welfare constraints, we often encounter a set of solution of single process operation like *dictatorship*, as pointed out by Aruka (1996). If we should find one of the most efficient activities and invest all the wealth into it, we can realize the maximal net income. This answer intuitively seems trivial. Hence, we may imagine that a much simpler system s with higher income often appears or be more probable, compared with a more complex system c:

$$Pr(\omega^{\text{higher}} | s) > Pr(\omega^{\text{higher}} | c)$$

On the other hand, as for complexity, it holds that

$$Pr(\omega^{\text{lower}} | s) > Pr(\omega^{\text{lower}} | c)$$

On average, it then holds that

$$\langle \omega^s \rangle > \langle \omega^c \rangle.$$

It is noted that a more complex production could lead to an increase of **the number of interactions** of nodes or productive activities, if we introduce a *recyclic* production of the economic system. In what follows, we adopt the next hypothesis on **hierarchical inclusion**:

A simpler subsystem with higher profitability could be integrated by means of complementation of a more complex subsystem.[6)]

3. Different Expectations by Heterogeneous Agents and the Effects of Neighbouring Agents

3.1 A traffic problem by type selection

According to Strang (1991, pp. 211–213), we have a kind of confusion about "expected" group size. Now suppose, for instance, that there were 10 cars given, with a single driver in three cars, and three people, including a driver, in the remaining seven cars. The expected class size *in view of the visitor* is:

the summation of 3 cars with a single person $\left\{1\left(\frac{1}{24}\right)+1\left(\frac{1}{24}\right)+1\left(\frac{1}{24}\right)\right\}$

+the summation of 7 cars with three persons $\left\{3\left(\frac{3}{24}\right)+\cdots+3\left(\frac{3}{24}\right)\right\}=2.75$

=2.75 persons in the car.

his average is just the average number of persons per car that a *random* visitor can expect. On the other hand, the average number of persons which a *random* city authority or a policy maker can expect is

$$\frac{1\times 3+3\times 7\,\text{(persons)}}{10\,\text{(cars)}}=2.4 \text{ per car.}$$

[6)] Aruka and Mimkes (2005), by the use of von Neumann-Sraffa model of production (von Neumann, 1937; Sraffa, 1960), attained to the next result:

Theorem 1. *The probability of multiple truncations compatible with a given rate of interest r must be augmented if the number of truncations increases in the range of $g \leq r$. Average welfare could then be risen by a hierarchical inclusion of a single process operation of higher profitability, i.e., by an increase of complexity.*

3.2 An expectation failure

A random visitor feels to be in a more crowded car than in the average car which the city authority estimates. In this setting, as Strang (1991, pp. 211–213) also points out, the next relation holds:

> Traffic problems could be eliminated by raising the average number of people per car to 2.5, or even 2. But that is virtually impossible. Part of the problem is the difference between (a) the percentage of cars with one person and (b) the percentage of people alone in a car. Percentage (b) is smaller. In practice, most people would be in crowded cars.

$$\text{(a)}\,\frac{3}{10} \gg \text{(b)}\,\frac{3}{24}$$

That is to say,[7] the difference between (a) and (b) gives a random visitor a dominant motivation to switch from being a fellow passenger to being a driver.

3.3 Effects of neighboring behavior

The above instance in *type selection* may all give a heuristic finding of *distortion* from the weighted mean of total size. We can enumerate other similar situations as we like. In a random state, whether in a traffic matter, or in a Prisoners' Dilemma game situation, every random participant may decisively depend on **the neighboring type sizes** to select her own choice of transitions, as long as the change of types cannot hurt his or her individualistic gain. The impulse for adaptation to another type could always be generated in our situation.

The motivation due to this kind of distortion to move may be observed by connecting with a kind of *passion within reason*.[8] Transition may be justified in view of his or her passion. Put another way, **a generation of transition** may have an individualistically rational ground.

3.4 The view of the Nash bargaining problem

We notice that our types cannot be "divisible." This world constitutes just the field of the

[7] We can easily prove the next proposition: With groups of sizes $x_1, x_2, \cdots, x_n$ adding to $G=\sum_i^n x_i$ the average size is $\frac{x_1}{G}$. The chance of an individual belonging to group 1 is $\frac{x_1}{G}$. The expected size of the group is $E(x)= x_1\frac{x_1}{G}+\cdots+x_n\frac{x_n}{G}$.

[8] Robert Frank loves to discuss plentiful discussions of this kind. See Frank (1988).

Nash bargaining problem indicated by Nash (1951). The types of car situation are 24 kinds in the above traffic problem, although the seating capacity of a sedan is normally limited. In fact, some truncation will be needed for a practical use. Each way of which type a random driver can take may be taken as a lottery.

Thus the total number of the lotteries is

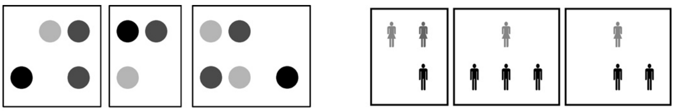
$$\sum_{r=1}^{24} \frac{n!}{r!(n-r)!} = 16,777,215 \text{ ways.}$$

Table 2. Size combinations.

Car	Size 1	Size 2	Size 23	Size 24
	${}_{24}C_1$	${}_{24}C_2$	${}_{24}C_{23}$	${}_{24}C_{24}$[a]

[a] ${}_nC_r = \frac{n!}{r!(n-r)!}$ is the binomial coefficient. ${}_{24}C_{24}$ is empty for a random driver, because there is not any single driver.

This gives us just the idea of distribution of n balls in r boxes $1, \cdots, n$ as leading to the idea of statistical mechanics. The first part (a) of Fig. 4 implies that there are three different coloured balls, three different sized boxes, while the number of the light gray ball is 4, the dark gray's is 5, and the black's is 3. In Fig. 4(b), we have two types of gender (*colour*) and three different subgroups (*box*), but the number of successful couples (*marriage*) of each gender never rises if it never permits *transition* of the members into another box.



(a) (b)

Fig. 4. Distribution of *r* balls in *n* boxes.

3.5 Our new focus on the interaction among heterogeneous agents with many finite numbers

In the Nash bargaining problem, the players are like two boys only allocating "indivisible stationery." They are not random players. Many agents never appear. In the case of featuring the "Nash bargaining solution," Nash thus assumes that the players' preference over lotteries obey the von Neumann-Morgenstern axioms and hence can be represented up to positive affine transformations by a pair of von Neumann-Morgenstern utility functions.[9)] We therefore anticipate that here is a certain branching point to go to a new theory of interaction. It must be noticed that Nash pertinently formulates the interaction among *just two boys*. Another way may be given from the view of interaction among *heterogeneous boys* with many finite numbers.

Given 16,777,215 ways, *a huge number* of possibilities, we may rather regard these respective events equally happening. One idea is to assume that all the possibilities could occur equally. This idea will lead to introducing an idea of *Gibbs distribution*, for instance. We thus rather focus on distribution, i.e., a macroscopic attribute.[10)]

3.6 The idea of statistical mechanics and the Boltzman distribution

We can now illustrate the Boltzman distribution in view of statistical mechanics. We furthermore assume not only that here are a set of N_k balls in the boxes k, but also that the boxes are endowed with volumes V_k and values E_k. We then have Fig. 5, such as a distribution of N balls in K boxes of different volume V_k and value E_k. Here, values E_k may be interpreted with energy or price.

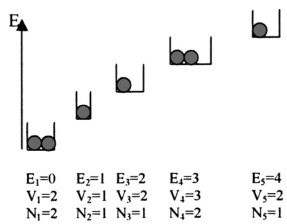


Fig. 5. Distribution of *N* balls in *K* boxes of different volume V_k and value E_k.

We denote by $\sum_k V_k$ by V, and $\sum_i N_i$ by N. We then construct the following Lagrange

[9)] Recently, "pessimistic subjectivity" is taken account of in the theory of expected utility theory. The theory of this type is called *Choquet* expected utility theory. But we are rather more interested in the interaction of heterogeneous agents. See Bassett W., R. Koenker, and G. Kordas (2004), for example.

[10)] Finally, we must notice *another point*. Even in a society where there are only 24 people, we are always faced with too many menu lists on type-size allocation when we must select. Suppose we are always given an initial allocation of types. We can then always hold the initial allocation as a *status quo* (standstill) policy. In the case of Nash bargaining, the allocation is called the *threat* or *impasse point*.

[11)] $\frac{\partial L}{\partial N_i}=0$ implies $E_i - T\ln\left(\frac{N}{N_i} - \ln\frac{V_i}{V}\right)=0$. Hence $\ln\frac{N}{N_i}\frac{V_i}{V}=-\frac{E_i}{T}$.

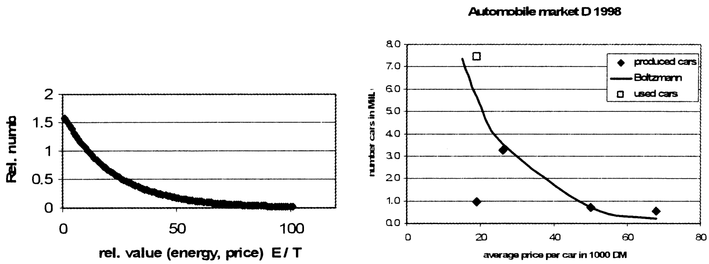


Fig. 6. Boltzman distribution.

Table 3. Automobile market in Germany 1998.

price in DM	19,000	26,000	50,000	68,000
produced units	940	3250	720	540
used cars	745			$\sum$=1,120

maximization problem with constraints:

$$L(N_i)=E_0+N_iE_i-T\{N\ln N-N_i(\ln N_i-\ln(V_i/V))\}\Rightarrow\text{max!}$$

Solving this problem[11)], it follows **the Boltzman equilibrium distribution**:

$$\frac{N}{N_i}=\frac{V_i}{V}\exp\frac{-E_i}{T}$$

This distribution exhibits a relationship between the relative value and the relative number. We produce **the Boltzman distribution** of the automobile market in Germany in 1998. Cars are graded by the level of price. We employed the figures of 4 different classes of sales price measured in DM: 19,000, 26,000, 50,000, and 68,000.

4. An Elementary Introduction of Evolutionary Theory to Social Interaction of Heterogeneous Agents

4.1 Stochastic systems with constraints

We have defined **entropy** and **temperature** and used them in an economic system as **complexity** and **welfare**. Now we apply a constraint to entropy, according to Lagrange, and can then study the stochastic systems with constraints.

It is noted that this formulation can always be activated in *cellular automata* to directly represent **the effects of neighbouring agents**, as Mimkes (2003) showed. Suppose we have a stochastic system of N_i *heterogeneous* elements:

L: Lagrange function.

$E(N_i)$: constraint.

T: Lagrange parameter.

P: probability of distribution.

P_E: probability of constraint.

We then have the maximization problem:

$$L=E(N_i)+T\times\ln P(N_i)\Rightarrow maximum!$$

Here we introduce the relative size of group i:

$$x_i=N_i/N$$

We then have the next relationships:

$$\ln P=-N(x_i \ln x_i)$$
$$E=Nx_i\varepsilon_i+\varepsilon_{ik}x_ix_k+\cdots \tag{1}$$

Expanding $E(x_iN)$ into a Taylor series, we obtain the last equation (1). We thus have the following maximization problem:

$$L=N\{x_i\varepsilon_i+\varepsilon_{ik}x_ix_k-T(x_i \ln x_i)\}\Rightarrow maximum!$$

4.2 The simplest case: Interaction between the binary agents

We illustrate the simplest case where there are only two heterogeneous agents called A and B. N_A is the number of A, and N_B is the number of B. In this case, we can put the efficient as follows:

$$E=N_Ap_AE_{AA}+N_Ap_BE_{AB}+N_Bp_AE_{BA}+N_Bp_BE_{BB}$$
$$p_{\mathrm{B}}=N_B/N=x$$
$$p_A=N_A/N=(1-x)$$

Rearranging the above relations,

$$\ln P=N\{-x\ln x-(1-x)\ln(1-x)\}$$
$$E(x)=N\{E_{AA}+x(E_{BB}-E_{AA})+(E_{AB}+E_{BA})-(E_{AA}+E_{BB})x(1-x)+\cdots\}$$

We put

$$\varepsilon=(E_{AB}+E_{BA})-(E_{AA}+E_{BB})$$

We then have the maximization problem in the binary interaction:

$$L=N[E_{AA}+x(E_{BB}-E_{AA})+\varepsilon x(1-x)-T\{x\ln x+(1-x)\ln(1-x)\}]\Rightarrow max!$$

Now, by the use of ε, we define the characteristics of binary interaction as follows:

Cooperation: $\varepsilon>0$.

Integration: $\varepsilon=0$.

Segregation: $\varepsilon<0$.

4.3 Six real structures in binary agent systems

We can rearrange the above problem as follows:

$$L(x, T)=L_0+\varepsilon x(1-x)-T\{x \ln x+(1-x) \ln(1-x)\} \Rightarrow \mathit{maximum}!$$

Here $\varepsilon=(E_{AB}+E_{BA})-(E_{AA}+E_{BB})$ is the **structure parameter** of phases.

The problem can easily be solved into:

$$\frac{T(x)}{\varepsilon}=\frac{1-2x}{\ln x-\ln(1-x)} \tag{2}$$

This solution can depict **the phase diagram** for binary alloys (e.g., *Au Pt*). According to the conditions of coefficient ε and E_{ij}, we now summarize such six real structures in binary agent systems as produced in Table 4.

4.4 An example: Intermarriage interaction

A binary relation can have the percentage dynamics

$$P(x)=2x(1-x) \tag{3}$$

Let x be the proportion of *minority*, for instance. If x approaches 0.5, its percentage must be 0.5; if it exceeds, it cannot by itself be a minority. We take the example of *intermarriage* in Europe, in particular, in Germany (*D*). If $T/|\varepsilon|>1$, **ideal integration** then dominates; if $T/|\varepsilon|<1$, **segregation** then dominates; if $T/|\varepsilon|\to 1$, **separation** then dominates; **Social Temperature** T is given by equation (2). Difference of temperature characterizes social state. If $T/|\varepsilon|$ is quite low, the social state must be aggressive since the separation of the population dominates.

Table 4. Six real structures in binary agent systems.

Structure parameter	Society	Nature
$\varepsilon<0$ and $E_{AB}+E_{BA}>0$	segregation	alloy, real liquid
$\varepsilon<0$ and $E_{AB}+E_{BA}<0$	aggression	chemical reaction
$\varepsilon>0$ and $E_{AB}=E_{BA}>0$	partnership	compound
$\varepsilon>0$ and $E_{AB}\neq E_{BA}>0$	heirarchy	solid
$\varepsilon=0$ and $E_{AB}+E_{BA}>0$	democracy	ideal liquid
$\varepsilon=0$ and $E_{AB}+E_{BA}=0$	global structure	gas

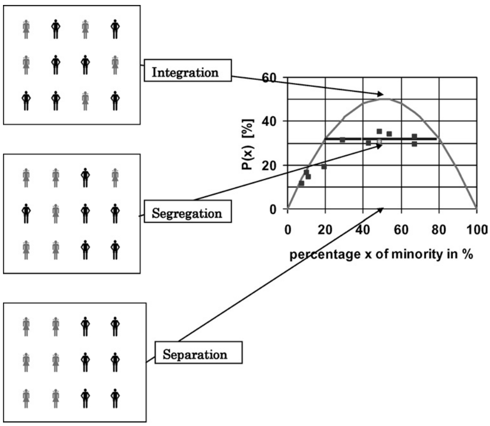


Fig. 7. Intermarriage as a thermometer of social temperature.

* The mark of Switzerland (*CH*: Confederation Helvetica) "●" is located approximately on the centre.

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