Elsevier Editorial System(tm) for Physica A
Manuscript Draft

Manuscript Number: PHYSA-D-09-00659R1

Title: Stokes Integral of Economic Growth The Laws of Calculus and Neoclassical Theory

Article Type: Research Paper

Section/Category: Econophysics

Keywords: Econophysics, 89.65.Gh, calculus, Stokes integral, differential forms, ex ante, ex post, production, economic growth

Corresponding Author: Dr. Jurgen Mimkes,

Corresponding Author's Institution: Paderborn University

First Author: Jurgen Mimkes

Order of Authors: Jurgen Mimkes

Abstract: Economic growth depends on capital and labor. Accordingly, two dimensional calculus has been applied to economic theory. This leads to Riemann and Stokes integrals of production and growth, to "ex ante" and "ex post" differential forms and to the first and second law of economics. In the first law, d M = d K - d P, growth (d M) depends on capital (d K) and production (d P). The second law confirms the existence of an "ex ante" production function (d F): d M = l d F. In contrast to the Solow model we find Y ^{1} F: "ex post" income (Y) cannot be equal to the "ex ante" production function (F), but requires an integrating factor (l). The production function cannot be chosen freely, F is unique for each economic system and is determined by probability (entropy), not by the Cobb Douglas function.

Suggested Reviewers:

List of 5 Possible Referees -

The manusscript is on econophysics, macro-economics based on methods from thermodynamics. The potential impact is hopefully a revision of neoclassical theory.

This paper may be well understood by physicists, as they are used to calculus, Stokes and Gauss integrals. Unfortunately, this paper is hard to read for economists, as they often do not know enough about calculus in two or three dimensions, including Stokes and Gauss. This has been an experience at many international conferences.

Here are full names and email addresses of five experts:
1. Rosario Mantegna mantegna@unipa.it
2. Peter Richmond peter_richmond@btinternet.com
3. Bikas Chakrabarti bikask.chakrabarti@saha.ac.in
4. Victor Yakovenko yakovenk@umd.edu
5. Yuji Aruka aruka@tamacc.chuo-u.ac.jp

Stokes Integral of Economic Growth

Calculus and the Solow Model

Jürgen Mimkes

Physics Department, Paderborn University,
D - 33096 Paderborn, Germany

Abstract

Economic growth depends on two parameters, capital and labor. Accordingly, two dimensional calculus has been applied to economic theory. This leads to Riemann and Stokes integrals of production and growth, to "ex ante" and "ex post" differential forms and to the first and second law of economics. In the first law, $\delta M = d K - \delta P$, growth ($\delta M$) depends on capital ($d K$) and production ($\delta P$). The second law confirms the existence of an "ex ante" production function ($d F$): $\delta M = \lambda \, d F$. In contrast to neoclassical models we find $Y \neq F$: "ex post" income (Y) cannot be equal to the "ex ante" production function (F), but requires an integrating factor (λ). The production function cannot be chosen freely, F is unique for each economic system and is determined by entropy, not by a Cobb Douglas function.

Keywords: Econophysics, 89.65.Gh, calculus, Stokes integral, differential forms, ex ante, ex post, production, economic growth

1. Introduction

1.1 Econophysics

Econophysics is the exchange of methods and ideas between natural and socio-economic sciences. The term "econophysics" was first introduced by E. Stanley in 1995 in finance [1] and has since developed into a world wide community of natural and socio-economic scientists working in finance, income distributions, economics and social sciences [2-15]. A good overview has been given recently by Yakovenko and Rosser [16].

This paper focuses on economics following an idea presented by E. T. Jaynes in 1980 [5]: „Classical economics was built largely on the analogy to mechanics. But a macroeconomic system is in some ways more like a thermodynamic system. Our theory is qualitatively wrong, because we have chosen the wrong set of variables." The present paper transfers calculus and experience in thermodynamics to production and economic growth and compares the results to neoclassical theory and the Solow model.

J. Mimkes, Tel. +495251 63180, e-mail: Juergen.Mimkes@uni-paderborn.de

1.2 Basic ideas of the Solow model of economic growth

(1). Economic growth in the Solow model [17-19] depends on income (Y) and consumption costs (C). The surplus (S)

$$S = Y - C = I \qquad \text{(i)}$$

is the difference of income and costs. In a closed economy the surplus is saved and reinvested, (I) is the investment.

(2). A production process is defined by a production function F, which exists for all production processes. F is the output of economic production.

(3). The first variable or production factor of economic production is given by capital (K). Capital includes circulating and fixed capital.

(4). The second production factor of economic production is given labor force (N). This leads to

$$F = F(K, N) \qquad \text{(ii).}$$

(5). Income (Y) is generated by production output (F):

$$Y = F(K, N) \qquad \text{(iii).}$$

(6). The Solow model assumes a Cobb Douglas production function F_{CD},

$$F_{CD} = A\ K^{\alpha}\ N^{1-\alpha} \qquad \text{(iv).}$$

The parameter A is a technology factor and α is called "elasticity", with the condition $0 < \alpha < 1$.

(7). The production output per capita (F / N) of a production process is

$$f_{CD} = \frac{F_{CD}}{N} = A \cdot \left(\frac{K}{N}\right)^{\alpha} = A \cdot k^{\alpha} \qquad \text{(v)}$$

(8). Production output per capita (f_{CD}) depends on capital per capita (k),

$$k = K / N \qquad \text{(vi).}$$

(9). The factor (A) of advancement of technology in Eq.(v) may be defined as a second production factor of productivity (f),

$$f = f_{CD}(k, A) \qquad \text{(vii)}$$

(10). In the Solow model the income per capita is given by $y = Y / N$. The relative growth rate of GDP per capita is $d\,y / y = d\,f_{CD} / f_{CD}$:

$$d\,y / y = \alpha\ d\,k / k + d\,A / A \qquad \text{(viii).}$$

Relative growth according to the Solow model depends on relative growth of capital (d k / k) and on relative advancement of technology (d A / A).

(11). Economic growth also depends on population growth. In the present paper the total number of laborers (N) is assumed to be constant.

Only these 11 points of the Solow model will be discussed in this paper, as they are fundamental assumption of neoclassical theory.

2. Calculus in two dimensions

The production functions F (K, N) in (ii) and f (k, A) in (vii) are functions of two parameters, but standard economic theory is based only on calculus in one dimension. What is the difference between one and two dimensional calculus? One dimensional calculus leads to exact differential forms and the path independent Riemann integral. In two dimensional space we obtain in addition not exact differential forms and the path dependent Stokes integral [20, 21]. In natural science these integrals lead to the specific laws of mechanics, magnetism, electricity and thermodynamics.

The same is true for economics: In two dimensional economics exact differential forms with path independent Riemann integrals may be called "ex ante" forms. Not exact differential forms with path dependent Stokes integrals may be called "ex post" forms. The course of a falling ball is an exact differential form, which may be integrated in advance (ex ante), before the ball arrives. In contrast the course of shares at the stock market is represented by not exact differential forms, which can only be determined afterwards (ex post), after the path of integration is known, when the shares have been sold. Also, tax refunds can only be filed after the year has ended. "Ex ante" and "ex post" are important issues in economics and finance.

2.1 "Ex ante" differential forms and the Riemann integral

The exact differential form d f

$$d f(x, y) = (\partial f / \partial x)\, d x + (\partial f / \partial y)\, d y$$
$$= a(x, y)\, d x + b(x, y)\, d y \qquad (1)$$

is marked by a "d" and has the following properties:

1. The mixed derivatives of "d f" will always be equal,

$$\partial^2 f / \partial y \partial x = \partial b / \partial x = \partial a / \partial y = \partial^2 f / \partial x \partial y \qquad (2).$$

2. Integrals of exact differential forms are called Riemann integrals, they depend on the integral limits A and B, but not on the path (u) of integration,

$$f(x, y) = \int_u df(x, y) \qquad (3).$$

d f may be called an "ex ante" differential form, it may be integrated without knowing the path of integration in advance, since f (x, y) is independent of the path.

3. The closed Riemann integral along a circle in the x – y plane will always be zero,

$$\oint df(x, y) = 0 \qquad (4).$$

The closed integral may be divided into two integrals from A = (x_0, y_0) to B = (x_1, y_1) and back from B to A:

$$\oint df(x,y) = \int_{x_0,y_0}^{x_1,y_1} df + \int_{x_1,y_1}^{x_0,y_0} df = \int_{x_0,y_0}^{x_1,y_1} df - \int_{x_0,y_0}^{x_1,y_1} df = 0 \quad (5).$$

The integrals will cancel, as the have the same limits but in opposite direction.

2.2 "Ex post" differential forms and the Stokes integral

The not exact differential form δ g,

$$\delta g(x, y) = a(x, y)\,dx + b(x, y)\,dy \quad (6)$$

is marked by a "δ" and has the following properties:

1. The mixed derivatives of δ g will not be equal,

$$\partial b / \partial x \neq \partial a / \partial y \neq \partial^2 f / \partial x \partial y \quad (7).$$

2. Integrals of not exact differential forms are called Riemann integrals, they depend on the integral limits A and B, and on the path (u) of integration,

$$g_u(x,y) = \int_u \delta g(x,y) \quad (8).$$

The differential δ g may be called an "ex post" differential form, it may only be integrated, after the specific path is known. A function g (x, y) does not exist, only functions g_u (x, y), which are different for each path (u).

3. The closed integral of δ g along a circle in the x – y plane is not zero,

$$\oint \delta g(x,y) \neq 0 \quad (9).$$

The closed integral may be divided into two integrals from A = (x_0, y_0) to B = (x_1, y_1) and back from B to A:

$$\oint \delta g(x,y) = \int_{x_0,y_0}^{x_1,y_1} \delta g + \int_{x_1,y_1}^{x_0,y_0} \delta g = \int_{x_0,y_0}^{x_1,y_1} \delta g - \int_{x_0,y_0}^{x_1,y_1} \delta g \neq 0 \quad (10).$$

The integrals along a circle in the x – y plane will not cancel. They have the same limits in opposite direction, but the path (u) along a circle is different.

3. Stokes integrals in economics

3.1 Quesnay´s model of a natural production circuit (δ P)

The French economists and physician Françoise Quesnay (1694 – 1774) has based the natural production circuit on the closed blood stream:
fig. 1 work is transferred from households to agriculture and consumption goods are brought back from agriculture to households. Consumption goods (produce) are the rewards of work or labor input.

The production circuit is an important step beyond linear production models. In physics work and consumption goods are both measured in energy units, in Joule, kWh or calories. For survival and growth the energy input at work must be less than consumption of food by workers at home. The energy balance $\Delta Q_u = Q_u$ (work) – Q_u (produce) < 0 must be negative,

$$\oint \delta P = \Delta Q_u \prec 0 \qquad (11).$$

Eq.(11) is a Stokes integral, as the closed integral of the production circuit is not zero. Production (δ P) is a not exact differential form and a first example of an "ex post" term: the output of production (P_u) can only be given at the end, and depends on the path (u) of integration, on the production process.

3.2 The Stoles integrals of modern production circuits (δ P)

Modern production circuits are more complex. Labor is still transferred from households to industry and consumption goods are sent from industry to households. But the consumption goods are no more the reward for labor input. There is a second monetary circuit (δ M), fig.2. Labor is paid by wages and consumption goods of industry are paid by consumption costs of households. In economics energy is not measured in Joules, but by a price in monetary units, in €, US $ or British £, the price may vary in time. An important example is the oil price.

The equivalence of production and monetary circuits corresponds to equilibrium of supply of producers (δ P) and demand of buyers, who pay the agreed amount of money (δ M).

In fig. 2 the monetary circuit (δ M) – dashed line – measures the production circuit (δ P) – solid line.

The equivalence of the two circuits at a given time may be expressed by

$$\oint \delta M = -\oint \delta P \qquad (12).$$

The equivalence of production and monetary circuits, Eq.(12) is the basis of the present economic theory, and is valid for households, firms, industries, countries, economies. The negative sign in Eq.(12) corresponds to the opposite direction of the Stokes integrals in fig. 2.

3.3 Income, profit, surplus (δ M)

Profit or surplus (S_H) of households is defined by the closed monetary Stokes integral of (δ M) in Eq.(12),

$$\oint \delta M = S_H \qquad (13).$$

The closed integral may be split into two parts, from industy (Ind) to households (H) and back from households to industry: into income (Y_H) of households paid by industry, and consumption costs of households (C_H), paid to industry.

$$\oint \delta M = \int_{Ind}^{H} \delta M + \int_{H}^{Ind} \delta M = Y_H - C_H = S_H \qquad (14).$$

$$\int_{Ind}^{H} \delta M = Y_H \qquad (15),$$

$$\int_{H}^{Ind} \delta M = -C_H \qquad (16).$$

Income and costs are defined by open path dependent integrals of (δ M) in Eqs.(15) and (16). Income, costs, and surplus are all given by a specific path of integration of the same (ex post) differential form (δ M).

S_H , Y_H , C_H are measured in units of money per cycle time, US \$ per hour, € per day, month or year, depending on the specific contracts of the partners.

A common example for the Stokes integral (14) is the monetary circuit of households. In classical economics Eq.(14) has been simplified by Eq.(ii)

$$S_H = Y_H - C_H \qquad (17),$$

as the Solow model does not employ Stokes integrals.

In fig. 3 the following example is given: Households of industrial workers may earn Y_H = 100 € per day. If they consume C_H = 90 € per day, households will save S_H = 10 € every day.

Fig. 3 has not been plotted as a circle, but as a rectangle. The Y – axis shows the money of households earned (Y_H) or spent (C_H) and corresponds to capital (K), the first production factor of growth. The X – axis is so far undefined, it should correspond to the second production factor. This will be discussed in more detail in chapter 5 and in fig. 6.

4. Differential laws in economics

4.1 The first law

The Stokes integrals Eq.(12) lead to the first (differential) law of economics,

$$\delta M = dK - \delta P \qquad (18).$$

The "ex post" differential forms of profit (δ M) and production (δ P) in Eq.(12) are equal, except for an "ex ante" differential form (d K), as the closed integral of an exact form (d K) is zero. The negative sign of (δ P) indicates the opposite direction of the cycles, that labor has to be invested in order to make money. d K may be interpreted by capital.

Profit (δ M) and production (δ P) are path dependent and have the dimension money per cycle. Capital (d K) is path independent and has the dimension "money" independent of the cycle. Within one cycle all three may be measured in US \$, € or other currencies.

Eq.(18) is the balance of an economic system and tells us what we already seem to know: Economic growth or profit (δ M) depends on two parameters or production factors, capital (d K) and labor (δ P). The first variable (d K) is in agreement with the Solow model in Eq.(iii). The second variable of economics is work, production or labor (δ P).

Eq. (18) may be compared to the first law of thermodynamics, the balance of heat (δ Q), energy (d E) and work (δ W): $\delta Q = dE - \delta W$. Accordingly, Eq.(18) may be called the first law of economics.

4.2 Capital (d K)

Capital (d K) is the first production factor, independent of cycle times. Capital includes circulating capital (money) and fixed capital (goods) that may be measured in units of US $, € etc. The farm is the capital of the farmer, the office the capital of a firm, the production plant is the capital of a company. By selling all goods may be transformed into money.

Selling goods and buying commodities corresponds to a bouncing ball, where kinetic energy is transformed into potential energy and back. The total energy (E) is constant, unless frictional forces are active. In the same way the total capital (K) is constant, stores will often give full refund for acquired commodities, if they are like new. Materials like gold may be used to store capital, but most commodities will lose value after use due to frictional forces.

4.3 Labor force (N)

The second variable of economic growth in Eq.(18) is production (δ P) or physical work. This is in contrast to the Solow model, were the second production factor is assumed to be labor force (N). But, of course, production (δ P) will generally be *proportional* to labor force (N).

N is a number which defines the size of the system. In economic systems N is the number of laborers, in societies the size of the population. N does not appear explicitly in the first law, but all three differential forms (δ P), (δ M), (d K) are proportional to N. A constant labor force N may be eliminated by dividing the first law by N and calculating all differential forms “per capita”. These differentials are generally given in small letters, like in Eqs.(v - viii).

4.4 The second law of economics

In calculus of two dimensions a not exact differential (δ M) may be turned into an exact differential form (d F) by an integrating factor (1 / λ),

$$dF = \delta M / \lambda \qquad (19).$$

$$\delta M = \lambda \, dF \qquad (20).$$

F is called production function of the economic system. Eq.(20) confirms the idea of economic theory, Eq.(ii), that a production function F exists for every economic system. Eq.(19) may be called second law of economics, it corresponds to the second law of equilibrium thermodynamics, $d S = \delta Q/T$.

4.5 The integrating factor (λ)

The integrating factor λ corresponds to temperature T in physical systems. Temperature is the mean kinetic energy per particle of a closed system. Similar to the energy-temperature relation $d E = \alpha N d T$ in physics we may assume a linear relation between capital (K), population (N) and λ,

$$d K = \alpha N d \lambda \qquad (21).$$

Capital (K) per capita (N) is given as $k = K / N$, and α is a dimensionless constant. Dividing by N we may write Eq.(21) as

$$d k = \alpha d \lambda \qquad (22).$$

λ is proportional to the mean circulating capital k per person and may be measured in units like € or US $ per person. In markets λ will be proportional to the mean money per customer, in societies λ will be the living standard, in economies the GDP per capita.

4.6 Production and production factors

The first and second laws (18) and (20) may be combined eliminating δ M,

$$\delta P = d K - \lambda d F \qquad (23).$$

Production (δ P) depends on capital (d K), on the production function (d F) and on the integrating factor λ. At constant parameters α and N we may replace capital (d K) by Eq.(21). We now find the two variables λ and F:

$$\delta P (\lambda, F) = \alpha N d \lambda - \lambda d F \qquad (24).$$

Production (δP) is a function of two production factors, λ and F.

The production factor λ is proportional to the mean capital (k), Eq.(22), the production factor in the Solow model. The second variable is given by the production function (F) or entropy of the economic system. F will be dimensionless, as λ is measured in units of money per person.

4.7 Production function and entropy (F)

What does entropy (F) mean in production? Entropy may be regarded as a measure of disorder, and (– d F) as ordering, the negative change of disorder. Production according to Eq.(23) may be defined by:

Production is generating capital (d K) and reducing disorder (-d F).

A worker at an automobile plant reduces the disorder of parts by putting the right parts together. A medical doctor brings our body into the right order, a scientist brings many ideas into the right order to create a theory. Ordering parts for a special product is the same in Europe or China, but due to the standard of living (λ) the price of labor (– λ d F) will be different.

The production function (F) in Eq.(24) may be calculated for zero profit ($\delta P = 0$),

$$dF = \alpha N d\lambda / \lambda = d N \ln(\lambda^{\alpha}) = d N \ln(k^{\alpha}) \qquad (25).$$

The entropy production F is given by a logarithmic Cobb Douglas function. The production function is dimensionless.

4.8 Example for entropy (F) of economic systems

Entropy is generally given by statistics. Like in statistical physics entropy (F) is defined by the elements of the economic system,

$$F = \ln \Omega \qquad (26).$$

Ω is the number of possibilities to place of N objects in K (price) classes:

$$\Omega = N! / (N_1 ! \ldots N_k !) \qquad (27).$$

At large values of N the Stirling formula $\ln N! = N \ln N - N$ may be applied,

$$F(N_k) = N \ln N - \Sigma N_k \ln N_k \qquad (28).$$

The entropy production function (F) is given as a dimensionless function, depending only on the number N_k of items in K different classes of the economic system:

a) The production function F of a market is given by the number N_k of pieces of K different commodities.
b) The production function F of a company is given by the number N_k of people in K different jobs.
c) The production function F of an economy is given by the number N_k of companies in K different fields of production.

Example: A company has N_1 permanent and N_2 temporary employees. The total number N of employees is constant. Fig. 4 shows the entropy roduction function

$$F(N_1, N_2) = (N_1 + N_2) \ln (N_1 + N_2) - N_1 \ln N_1 - N_2 \ln N_2 \qquad (29)$$

and fig. 5 the Cobb Douglas function [18]

$$U(N_1, N_1) = N_1^{\alpha} N_2^{1-\alpha} \qquad (30)$$

plotted versus the number N_1 of permanent employees in the range from 0 to 10. The parameter in both figures is the number N_2 of temporary employees in the range from 0 to 10.

All functions in figs.(4) and (5) show marginal growth, as is observed in economics. The entropic production function F in Eq.(29) is larger than the Cobb Douglas function U in Eq.(30) by a factor of about 1,4 for nearly all values of α. Entropy F is clearly the better production function. In addition F is independent of any arbitrary elasticity parameter α.

The result agrees with the work of Georgescu-Roegen [4] and Jaynes [5], who have introduced the idea of maximum entropy under constraints into economic theory. Maximum entropy is a well known topic in economics since, but is still rejected by main stream economists.

5. Carnot process of monetary Stokes integrals

Income (δ Y) and costs (δ C) depend on the production factors λ and F. According to Eqs.(15) and (16) we now obtain

$$\delta Y(\lambda, F) = \lambda \, dF \qquad (31).$$

$$\delta C(\lambda, F) = -\lambda \, dF \qquad (32).$$

In production entropy change refers to ordering and disordering of parts, in monetary circuits entropy change refers to collecting ($\Delta F < 0$) and distributing ($\Delta F > 0$) of money in coins or bills. Payments and earnings are closely related to the entropy of money distribution. The parameter λ indicates the monetary level, exchange rate or interest rate.

Example: A Chinese worker may get paid 100 yuan bills per hour, a manager of a US firm in Beijing may get 100 US $ bills per hour. λ is the exchange rate of the bills. λ may also be an interest rate: savers may earn an interest rate of 5 % per year, investors may have to pay 15 % per year.

5.1 The monetary Carnot cycle of production and trade

We may now discuss the monetary circuit of households in fig. 3 again. The surplus in Eq.(14) may be calculated in the λ – F plane along constant λ and at constant F,

$$S_H = Y_H - C_H = \oint \delta M = \oint \lambda \cdot dF = \Delta\lambda \cdot \Delta F \qquad (33).$$

Profit or surplus (S_H) is given by the product of the two production factors λ and F, the change in mean capital level (Δ λ) and change in entropy (Δ F). Following the idea of Carnot, the Stokes integral (33) may be carried out at constant λ and constant F. In fig. 6 the closed Stokes integral (33) is carried out from 1 → 2 →3 → 4 → 1.

Fig. 6 explains the profit S_H of households manufacturing and trading cars or other commodities in a country with lower standard of living (λ_1) like China and selling them in a country with higher standard of living (λ_2) like USA. Δλ is the change in standard of living bringing cars from China to the US market. $\Delta F < 0$ is the negative change of entropy and means collection of money from customers at the US market. $\Delta F > 0$ corresponds to distribution of money to workers in China. Fig. 6 corresponds to fig. 3.

1 → 2: US importers *collect* ($\Delta F < 0$) the amount $Y_2 = \lambda_2 \Delta F$ in US $ bills (λ_2) from customers at US automobile markets for imported Chinese cars.
2 → 3: US car importers *transfer* ($\Delta F = 0$) a certain amount of US $ bills from the US to Chinese car producers.

3 → 4: Chinese car producers *distribute* ($\Delta F > 0$) wages $Y_1 = \lambda_1 \Delta F$ in CNY bills (λ_1) to Chinese workers.
4 → 1: Chinese people *transfer* ($\Delta F = 0$) the US $, which had been brought to China, back to USA in exchange for US products. The monetary cycle is closed. The center area ($S_H = \Delta\lambda \Delta F$) in fig. 6 is the common surplus of Chinese and US trade. Cycle times depend on the contracts of the partners.

5.2 Carnot machines

Monetary and production circuits correspond to physical Carnot machines, they follow the same equations, the first and second laws:

The production circuit corresponds to motors and generators. They run on fuel from outside, in fact moters and production plants use the same fuel: oil! A motor requires two temperature levels, which are generally obtained by cooling one side. The efficiency of a motor rises with the temperature difference. If both sides have the same temperature, the motor will stop. The monetary circuit is equivalent to heat pumps or refrigerators. Work input leads to two different temperatures.

Through heat pumps a warm house gets warmer, the cold river gets colder. In refrigerators the inside gets colder and the outside warmer.

These results may be compared to companies, business and trade:

1\. a. A heat pump requires little energy to draw heat $Q_1 = T_1 \Delta S$ from a cold river and to deliver heat $Q_2 = T_2 \Delta S$ to a warm house with high efficiency.

1\. b. An import company pays low wages $C_\lambda = \lambda_1 \Delta F$ for production in a poor country and receives a high income $Y_\lambda = \lambda_2 \Delta F$ from sales in a rich country with high efficiency. Households, farms, business, companies, banks, factories and economies are capital pumps!

2\. a. A refrigerator starts creating cold inside and warm outside immediately, after it is plugged in. But we have to close the door and separate inside and outside. The larger the difference of temperatures, the higher is the efficiency of the refrigerator.

2\. b. When a company starts producing, it will automatically create a richer and a poorer side, capital and labor. In order to make the system work capital and labor have to be separated. The economic gap between capital and labor will grow with time. The larger the gap, the higher the efficiency.

3\. a. Motors run on fuel and oxygen from outside. They require two temperatures. The outside temperature may be defined by cooling agents. The motor will get hot inside, but the temperature is limited by conduction of heat. At equilibrium the motor runs at two constant temperatures.

3\. b. Factories run on external oil and resources for manufacturing. In order to run profitable they need to optimize profits by paying wages as low as possible and selling their products as expensive as possible. High competiton in producing, selling and buying acts as a cooling agent and will determine the equilibrium price and wage levels.

4\. a. The cooling systems of a motor must always be checked. If the cooling system is defect, it needs to be repaired by a technician.

4. b. Trusts immediately lead to highest prices, the mechanism of competition has broken down. Unemployment immediately leads to lowest wages. The mechanism of competition has broken down. *In a well working economy the government must protect the efficient market by enforcing proper antitrust laws and invest in education to keep unemployment low.*
5. a. Motors, pumps have periodic cycles, the time depends on construction.
5. b. Economies have periodic (day, month, year) or not periodic cycles.

We may now discuss the economic circuit of households in fig. 3 on the basis of the Carnot process. In fig. 3 wages seem to flow from industry to households, consumption costs seem to flow from households to industry like in a closed blood system. But this interpretation is not correct. In a human body the blood is indeed conserved as carrier of energy, but the energy, which is needed for survival, is not conserved in the blood circuit. The closed monetary Stokes integral may not be misinterpreted as conservative flux of money. In the human body, like in a motor, energy comes from an external flux of fuel and oxygen we absorbe every minute. This keeps the body running like a motor. In economic systems we have a constant influx of capital from external energy and raw materials, we produce at every moment. This influx from outside leeds to a growing or decreasing economy.

5.3 Banks as capital pumps

1. The first and second laws, Eqs.(18) and (20) may be considered as bank rules,

$$\delta M = dK - \delta P = \lambda\, dF \qquad (34).$$

Bank profits (δ M) may come from capital (d K) and production (δ P). But according to calculus (12) we find

$$\oint \delta K = 0 \qquad (35).$$

The closed integral of the exact differential form d K is zero, capital will not create capital. Investing in capital (d K) will not lead to profits in the long run. The probabilities for gaining and loosing is symmetric. Investing in capital means gambling, capital can only be redistributed. Profits (δ M) can only come from production (δ P).

Example: Banks try to persuade customers to invest into their winning strategies, which correpond to strategies in roulette. Betting a constant stake only on "even" or "red" and doubling the stake after each loss looks like a permanent winning strategy. But at any moment probability may force the player to pay the complete limited stock for doubling, and the player may be bancupt. The probability for the event of bancrupcy depends on the relation of stake and stock of the player. A higher stake means more greed and a shorter time (τ) to bancrupcy. In the financial world this time (τ) was about 80 years, the time between 1929 and 2009. Banks and savers should only

invest in production and the long term stock market. Revenues are lower, but so are the risks.
2. The risk may be calculated from the second law (20). Banks may be regarded as capital pumps: Banks pay interest ($C_\lambda = \lambda_1 \Delta F$) for saving acounts at low interest rates (λ_1) and high security (ΔF), and collect interest ($Y_\lambda = \lambda_2 \Delta F$) from investors at high interest rates (λ_2) and lower security (ΔF) from investors. A portfolio with a large number of assets has a high probability or high entropy to survive. Entropy (ΔF) is a measure of security, and the Carnot cycle in fig. 6 and Eq.(34) leads to the fundamental law of banking,

$$S_p = \oint \delta M = -\oint \delta P = \Delta\lambda \cdot \Delta F \qquad (36).$$

The product of positive returns ($\Delta \lambda$) and positive security (ΔF) is constant and given by the profit or surplus S_P of the company, portfolio or investment. The law (36) means: high returns, low security and vice versa. Banks prefer the term "risk " (R), the inverse of security (ΔF):

$$R \propto 1 / \Delta F \propto \Delta\lambda \qquad (37),$$

high returns $\Delta\lambda$, high risk R, and low returns $\Delta\lambda$, low risk R. This old bank rule is equivalent to the second law.

6. Economic growth

1. The Carnot circuit of a motor requires two temperatures, inside and outside. The heat that is created in every circuit, will dissipate to the inside and outside. The distribution of heat will influence the efficiency of the motor. If the inside gets hotter and hotter at constant outside temperature (due to a cooling system), the efficiency will grow quickly. If the outside runs hot (due to a failure of the cooling system), the motor stops.
2. The monetary circuit creates two groups of people in farms, companies, business firms: farmer and laborers, owner and workers, capital and labor, first and third world, rich and poor. Both groups together form the economic system. Accordingly, both groups will have to agree, how to divide the net output of each circuit. This is negotiated periodically by workers and employers, by unions and industry, by world trade conferences. In economic theory this is generally treated by game theory.

In a Carnot process the lower/cooler/poorer side (employees) will obtain the fraction (p) of the net output, the higher/ hotter/richer side (employer) will get the fraction (1 - p). If both groups reinvest their fraction of profit $p(Y_2 - Y_1)$ and $(1- p) (Y_2 - Y_1)$ after each cycle, they will grow in time (t). We obtain two equations for the two sides of the system, Y_1 (t) and Y_2 (t):

$$d Y_1 (t) = p \, (Y_2 - Y_1) \, d t \qquad (38)$$
$$d Y_2 (t) = (1- p) (Y_2 - Y_1) \, d t \qquad (39).$$

For $p \neq ½$ the solution of this set of differential equations is given by:

$$Y_1(t) = Y_{10} + p\,[Y_{20} - Y_{10}]\,[\exp((1-2p)t) - 1]\,/(1-2p) \qquad (40)$$
$$Y_2(t) = Y_{20} + (1-p)\,[Y_{20} - Y_{10}]\,[\exp(1-2p)\,t) - 1]\,/(1-2p) \qquad (41).$$

For $p = ½$ the solution is given by

$$Y_1(t) = Y_{10} + ½\,[Y_{20} - Y_{10}]\,t \qquad (42).$$
$$Y_2(t) = Y_{20} + ½\,[Y_{20} - Y_{10}]\,t \qquad (43).$$

The equations may be applied to all interdependent systems, to workers and employers, unions and industry, or interdependent countries. The results are presented in fig. 6.
For $p < 0,50$ economic growth is exponential. Surprisingly a percentage of $p = 10$ % of the profit (solid lines in fig. 6) is in the long run more profitable for the worker than $p = 25$ % (dotted lines).
The second surprise is the fair deal, 50:50, represented by dashed lines in Fig.7. This distribution of profits leads only to linear growth and is the least attractive distribution between two partners.
Fig 8 shows economic growth in China and USA [9] between 1990 and 2005. The difference in GDP per capita (Y_1) and (Y_2) is the basis for the economic motor between China and all western countries (USA).
Similar exponential growth has been observed in Germany and Japan after World War II, when trade with USA and other western countries led to exponential growth in both countries.
The efficiency (η) of US – Chinese trade relations for any labor intensive product corresponds to the efficiency of heat pumps, and is determined by the standard of living (λ_2) in USA and (λ_1) in China,

$$\eta_{ideal} = \frac{Y_2 - Y_1}{Y_1} = \frac{\lambda_2 - \lambda_1}{\lambda_1} \qquad (44)$$

In USA the GDP per capita is $\lambda_2 = 43.000$ US \$ per capita, and in China it is about 2.000 US \$ in 2006. The ideal efficiency is about $\eta_{ideal} = 20$ dollar for each dollar invested into Chinese labor intensive products. The real efficiency – like in heatpumps – is much lower, but still way above $\eta = 1$. This result is in contrast to US – Japanese trade. Due to similar standards of living in Japan and USA the efficiency of US – Japanese trade is close to an efficiency of $\eta = 1$.

Conclusion

The results of calculus based economics differ in many aspects from the Solow model of neoclassical theory. We may discuss the Solow model of the first chapter point by point:

(1). The basis of economic growth, Eq.(i), is a simplified version of the monetary Stokes integral, Eq.(14). But only the application of Stokes integrals leads to the laws of calculus based economics.
(2). Classical economics assumes the existence of an "ex ante" production function F, which depends only on production factors. This assumption has been confirmed by the second law, Eq.(19), which explicitly state the existence of an "ex ante" production function.
(3). The first variable or production factor of the production function F is capital (K). This is confirmed by calculus.
(4). The second production factor is not labor force (N), but production (δ P) or labor performed by the labor force.
(5). A basic assumption of the Solow model is the equivalence of income (Y) and production output (F): Y = F in Eq.(iii). *This assumption of the Solow model is not correct.* Income (δ Y) is "ex post" and the production function (d F) is "ex ante", both cannot be equal. According to the second law (20) income and production factor can only be connected by an integrating factor, $\delta Y = \lambda \, d F$.
(6). The Solow model is based on the Cobb Douglas function F_{CD}, Eq.(iv). *According to the second law (20) the Cobb Douglas function F_{CD} is not the optimal production function.* The optimal production function is given by the entropy of the economic system. Both functions look similar, but the properties of the functions are quite different.
(7). In the same way the Cobb Douglas function f_{CD} per capita must be replaced by entropy per capita f = F / N.
(8). The first variable or production factor of the function f is capital (k) per capita. However, in calculus based economics the integrating factor (λ), which is proportional to (k), is applied.
(9). Advancement of technology of the Solow model must be replaced by the production function F.
(10) Relative growth of production in economies in the Solow model

$$d y / y \;=\; d f / f \;=\; \alpha \, d k / k \;+\; d A / A \qquad \text{(viii)}$$

must be replaced by Eq.(24), as income is an "ex post" term, d y and y do not exist, (only δ y and y_u). With $\delta p = \delta P / N$ we obtain

$$\delta p / \lambda \;=\; \alpha \, d\lambda / \lambda \;-\; d F / N \;=\; \alpha \, d k / k \;-\; d f \qquad (45).$$

Relative growth model depends on relative growth of capital (d k / k) and on reduction of production disorder (- d f). *Advancement of technology (A) is not confirmed as a production factor*. The undefined term (A) must be replaced by the well defined entropy function (F or f = F / N). We may conclude:
1. The Solow model is only an approximation with respect to capital (k) and gives little information about the second production factor. The Solow model is not well suited for economic growth. This result is in agreement with many economists, who are critical of the Solow model and neoclassical theory [4-10].

2. Calculus based economics leads to the production factors mean capital k or λ and entropy F. Calculus is the appropriate tool for all problems of economics and economic growth and leads to reasonable models and exact results. Entropy is the proper production function. This is in agreement with E. T. Jaynes and others [4-6] working with maximum entropy.

Literature

1. Mantegna, R. N. and Stanley, H. E. *Introduction to Econophysics. Correlations and Complexity in Finance*. Cambridge University Press (2000)

2. Econophysics & Sociophysics: Trends & Perspectives
Bikas K. Chakrabarti, Anirban Chakraborti, Arnab Chatterjee (Eds.)
WILEY-VCH Verlag, Weinheim, Germany (2006)

3. Econophysics of Wealth Distribution,
A. Chatterjee, S. Yarlagadda, B.K. Chakrabarti, (Eds.)
Springer Italia (2005)

4. Georgescu-Roegen, N., *The entropy law and the economic process,* Cambridge, Mass. Harvard Univ. Press, (1974)

5. Jaynes, E. T. "*The Minimum Entropy Production Principle*"
Annual Review of Physical Chemistry, Vol. **31**, pp. 579-601, (1980).

6. Foley, D. K. “A Statistical Equilibrium Theory of Markets.” *Journal of Economic Theory* **62**, 321-345 (1994.).

7. Gallegatti M., Keen *S.,* Lux T. and Ormerod P., *Worrying trends in econophysics*, Physica A, 2006, 370

8. Lux T., Samanidou E., Zschischang E. and Stauffer D.,
"*Agent-Based Models of Financial Markets*",
Reports on Progress in Physics 70, 2007, 409 - 450

9. Keen S., *Debunking Economics*, Pluto Press, Sydney, 2001,

10. Rosser, J. Barkley, Jr.. “Econophysics.” In *The New Palgrave Dictionary of Economics,* 2nd edition, edited by L. E. Blume and S. N. Durlauf. London: Macmillan (2006)

11. ArukaY. and Mimkes J., “Complexity and Interaction of Productive Sub-systems under Thermodynamical Viewpoints,
Evolutionary and Institutional Economics Review, Vol. 2 (2005) , No. 2 pp.145-160

12. Dragulescu A., and Yakovenko V. M., "Exponential and power-law probability distributions of wealth and income in the United Kingdom and the United States",Physica A 299, 13 – 221 (2001).

13. Solomon, S, Richmond, P., *Stability of Pareto-Zipf Law in Non-stationary Economies*, In '*Economics with Heterogeneous Interacting Agents'*, Ed.:A. Kirman, J. –B. Zimmermann, Springer-Verlag, Lecture notes in Economics and Mathematical Systems 503, Berlin 2001 141-159

14. Weidlich W, *Sociodynamics*, Amsterdam : Harwood Acad. Publ., 2000.

15. Mimkes J, Binary Alloys as a Model for Multicultural Society , J. Thermal Anal. 43 (1995)

16. Yakovenko V. M. and Rosser J. B. *Colloquium: Statistical Mechanics of Money, Wealth and Income, arXiv:0905.1518*

17. Solow, A R. M., Contribution to the Theory of Economic Growth, The Quarterly Journal of Economics, 70 (Feb. 1956) 65-94

18. Cobb, C. W. and Douglas, P. H. : „A Theory of Production“ in American Economic Review, Mar 28 Supplement, 18 (1928) 139-165

19. Barro, R. J. and Sala–i–Martin, X., *Economic Growth*, New York: McGraw-Hill, 1995. 2. Ed. MIT Press, Cambridge, Mass. (2004)

20. Cartan, H., *Differential Forms,* Dover Publications; Translatio edition (2006)

21. Flanders, H., *Differential Forms with Applications to the Physical Sciences*, Dover Publications Inc. (1990)

22. www.econstats.com/weo/V014.htm

Figures

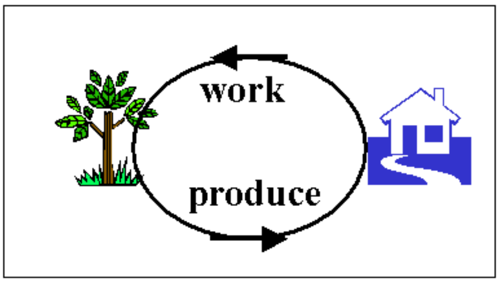

Fig. 1.

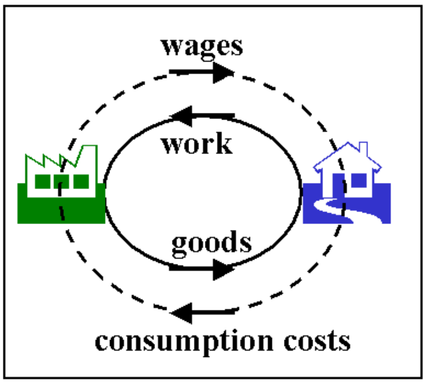

Fig. 2.

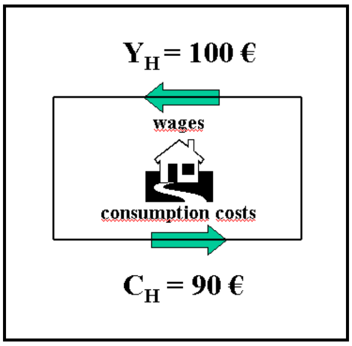

Fig. 3

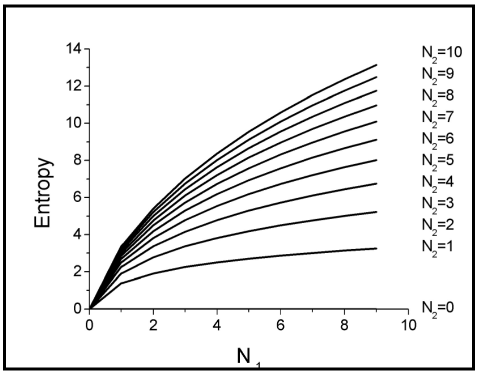

Fig. 4

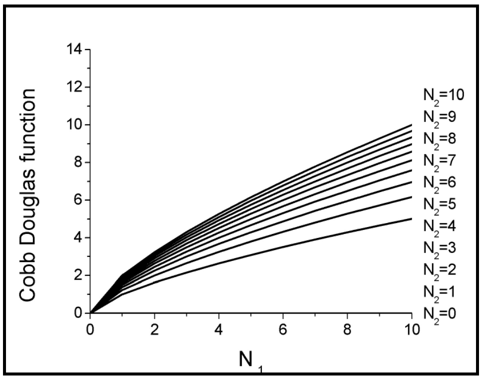

Fig. 5

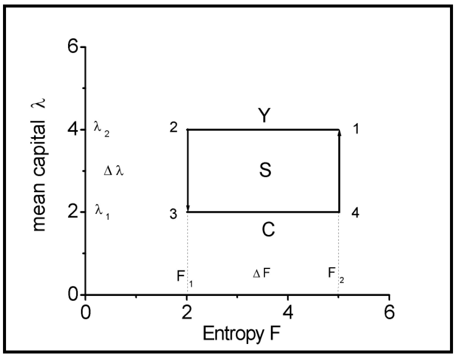

Fig. 6

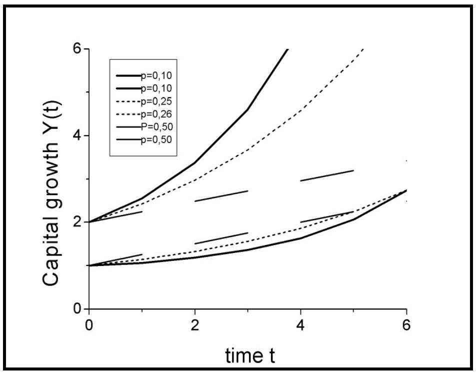
p=0,10
p=0,10
p=0,25
p=0,26
P=0,50
p=0,50
Capital growth Y(t)
time t
0
2
4
6
0
2
4
6

Fig. 7.

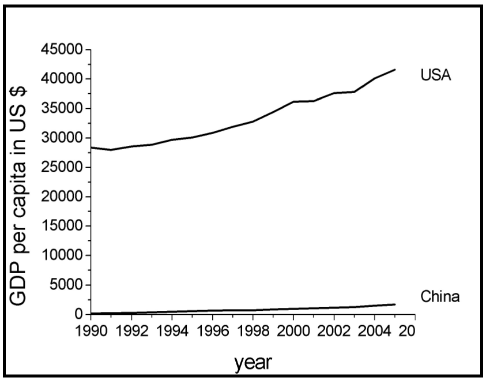
USA
China
GDP per capita in US $
year
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
1990
1992
1994
1996
1998
2000
2002
2004
20

Fig. 8.

Figure captions

Fig. 1. Quesnay´s model of a simple production circuit of a natural economy. Work of laborers is transferred from households to agriculture. In return produce is brought as a reward from agriculture to households.

Fig. 2. The model of a modern, complex economy contains two circuits or Stokes integrals, a production circuit (δ P) (solid line), and a monetary (δ M) or financial circuit (dashed line). The monetary circuit measures the production circuit.

Fig. 3 shows the monetary Stokes integral of households. The circuit may be split into income (Y_H = 100 €/day) of households, which is paid by industry to households, and consumption costs (C_H = 90 €/day) of households, which are paid by households to industry. The surplus of households is S_H = 10 €/day.

Fig. 4 shows the new entropy production function of a company with permanent and temporary employees, Eq.(29) is plotted versus the number of permanent employees N_1 in the range from 0 to 10. The number of temporary employees N_2 is also in the range from 0 to 10.

Fig. 5 shows the Cobb Douglas function, Eq.(30), which is plotted versus the number of permanent employees N_1 in the range from 0 to 10. The number of temporary employees N_2 is in the range from 0 to 10. The parameter of elasticity is chosen as $\alpha = 0,7$.

Fig. 6 shows the Stokes integral of the monetary circuit, Eq.(14), after Carnot in the $\lambda - F$ plane. $1 \rightarrow 2$: Households collect a high level income from a company selling in USA. $3 \rightarrow 4$: Households spend for consumption cost at a low level in China. Fig. 6 corresponds to fig 3.

Fig. 7. Economic growth of a system of two parties (capital and labor) according to Eqs.(40) to (43). One partner (Y_1) receives p percent of the surplus, the other partner (Y_2) receives (1-p) percent. For p = 0,10 we obtain high exponential growth, for p = 0,25 exponential growth is less. For p = 0,50 only linear growth is obtained.

Fig. 8. Economic growth of a system of two parties (China and USA). The difference in GDP per capita (Y_1) and (Y_2) is the basis of the economic motor between China and all western countries (USA) [21].

Ms. Ref. No.: PHYSA-D-09-00659
Title: Stokes Integral of Economic Growth The Laws of Calculus and Neoclassical Theory
Physica A

Dear Dr. Stanley,

thank you for sending three reviewers comments. All were very helpful and one reviewer was especially detailed in his comments. I am very grateful for the work invested by these referees.

I have attached the revised manuscript and a list of changes or rebuttals. I feel that this has indeed improved the paper, but I am always very open for further discussion.

Sincerely yours,

Juergen Mimkes

List of changes

(Reviers comments are given in italic and smaller size)

Reviewer #1: *This paper gives an interesting formulation of production laws and of economic growth using the analogy with classical thermodynamics with the important identifications of exact and inexact differentials. However, the paper needs some revision before it can be accepted for publication:*

a) With the identification of average money in the market (here GDP?) as the equvalent of temperature, established already (see e.g., arXiv:0905.1518 & refs. [2-3]), one should compare the estimated Carnot efficiency of the production processes in, say, China for cars and their sales in the US. There should be some attempt to compare the company data if possible.

1. The integrating factor (λ) corresponding to temperature (T) depends on the closed system. λ is proportional to capital per person ($\lambda = \alpha$ k) in markets and stands for standard of living in societies. This definition and the reference have been added to the paper.
2. The Carnot efficiency of Chinese USA trade has been added in the last chapter on economic growth.
3. The ideal efficiency applies only to labor costs, not to materials, which are traded wordwide at nearly the same price. The real efficiency of (car) production – like in heatpumps – depends on technical details like percentage of manual work, shipping costs, Chinese participation in profits, and cannot be compared to company data, yet.

b) The Fig. 5 shows some growths with time using dotted lines, dashed lines etc. -- none are explaned in the text or in the caption.

The dotted and dashed lines are now explained in text.

c) Also there are typos like the "THe negative sgn of indicates ... " in the para above eqn (13).

The typos have been corrected as much as possible.

Reviewer #2: *This is an interesting manuscript. I feel however it could be improved and sharpened.*
ET Jaynes, for example, published some time ago the idea that economics should include 'entropy' and illustrated how this concept would change the neo-classical theory of economics. He did not develop a formal calculus in the manner of this paper nevertheless I think this work ought to be mentioned. I have not made a serious search for other work but perhaps the author should indicate that he has.

I am grateful for mentioning ET Jaynes, I have cited some of his views in my introduction. For shortness I had only cited Georgecu-Roegen, but, of course, other authors are also active in maximum entropy applications in economics.

Some statements I feel confuse rather than clarify. For example:

Page 5:
'in classical economics 'lambda' corresponds to the efficient market hypothesis which leads to a single price level for a commodity'. Some additional explanation would help the reader here.

Indeed the idea of the efficient market hypothesis may be confusing. I still think it is correct, but I left out the term. “lambda” is proportional to the mean circulating capital per person in the system.

Page 6:
The section headed The variables of economic theory..... I feel this section could be clearer. As I understand the text, the author is simply developing an illustration along the lines of an ideal gas of economics and introducing a simple equation of state which leads to a specific form for the entropy function. This then equates to the logarithm of a Cobb Douglas function.

I have followed the referees advice and dropped the pressure part, as I do not use it any further in the paper. The result of a logarithmic Cobb Douglas function is valid only for zero production ($\delta P = 0$). For clarification I have added a more general example of entropy.

Page 7:
The author asks: what can a calculus based theory do better than neoclassical theory? Surely there are better answers than saying 'the parameters of economics are better defined'. Does this approach not introduce essentially new parameters that lead to a complete and self-consistent theory unlike the neoclassical theory which only applies at the equilibrium point but is incapable of dealing with even the smallest fluctuation? (The publications of economist Steve Keen might help the author develop some stronger statements.)

As a physicist I feel I cannot make as strong statements on neoclassical theory as a properly educated economist can. I have cited Steve Keen, as I think my results are supported by his views on neoclassical theory.

Page 9:
the author asserts: 'In a well working economy, the government must protect the efficient market by enforcing proper antitrust rules and invest in education to keep unemployment low.' This statement seems to say more about the political beliefs of the author rather than being something that emerges from his theory - or at least so it seems to me.

I have tried to show that government activity is important for the economic Carnot process, like the mechanics is for the motor (even though mechanics are not part of the equations).

Reviewer #3: *The paper studies a very interesting topic: an application of thermodynamics-inspired multivariable calculus to economic variables, such as money and productions. While the presented ideas look intriguing and stimulating, and I am sympathetic to this direction, the overall presentation and formalism in the paper suffer from poorly defined variables and unresolved conceptual issues. It produces impression of a half-baked theory. Thus, I hesitate to recommend the paper for publication in the present form. Perhaps, the author can improve presentation in a revised version by trying to address some of the issues raised below. The issues are listed in the order of appearance in the manuscript, not in the order of importance.*

1. In the first paragraph, the author introduces the terms "ex ante" and "ex post" without explaining their meaning in the economic context. This leave a reader confused right from the beginning of the paper. Literally, "ex ante" and "ex post" mean "before" and "after", but what does it mean in economics? Before and after what? The paper is submitted to a physics journal, so the author should not assume that the readers are familiar with economics jargon, and he should give explanations of economics terminology.

I have added an explanation for “ex ante” and “ex post”.

Throughout the paper, the author argues a lot with the neoclassical theory by Solow. However, what is that theory exactly, is not explained consistently anywhere in the paper (there are just scattered bits here and there). It would help the readers of Physica A if the author spent some dedicated space in the beginning of the paper to explain coherently what Solow's theory is.

I have added a paragraph with some important equations of the Solow model.

2. In the Introduction, the authors says that economic theory depends on two parameters, capital and labor. Then, some variables K and N appear in the paper, and only several pages later it is casually mentioned that K denotes capital and N denotes labor. The mathematical variables K and N must be defined from the very beginning, when the terms capital and labor first appear in Introduction.

This is correct and I have tried to define and explain the variables when they appear.

The meaning and the units of all variable in the paper must be clearly defined. It seems that labor N is the number of people in the workforce, so it is a dimensionless number. The meaning of capital K is never explained in the paper, particularly how capital K is different from money M.

The meaning of the number N and capital K have been stated in new chapters. Money is circulating capital, goods are related to fixed capital .

One of the central unexplained questions in the paper is the meaning of the closed path in Eq. (7). Does the closed path mean that labor N first increases and then decreases? Is this supposed to represent a demographic cycle: increase and decrease of population? Or is the change of N due to decreasing and increasing unemployment, so the cycle represents a business cycle (boom and bust)? What is the cycle for capital K? Does is mean borrowing money first and then repaying debt or what? Capital K appears for the first time in Eq. (12), but the meaning of K is not explained there.

The closed path has been explained by the Carnot cycle of car production in China.
In a motor the cycle is given by alternating changes of temperatur T and entropy S.
In trade the closed cycle is given by alternating changes of price or value levels λ and entropy F: buying (Δ F < 0) and selling (Δ F > 0), or distributing (Δ F > 0) and collecting (Δ F < 0).

The author claims that his main improvement over the traditional economic theory is a careful application of multi-dimensional (e.g, two-dimenstional) calculus, but he does not explain the meaning of his multi-dimensional variables and the meaning of a closed cycle!

3. There are problems with dimensionality of mathematical variables throughout the paper. Around Eq. (7), the author says that production P is measured in energy units (i.e., Joles), and around Eq. (8) he says that money M is measured in monetary units (i.e., dollars). Then, Eq. (8) is meaningless, because the dimensionalities of the right and left sides do not match.

The dimensions have been clarified. In physics energy is measured in Joules. In economics energy is measured in US $. (A well known example is the oil price.)

Moreover, Eqs. (10) and (11) relate money M to income and spending. However, income and spending are usually defined as influx and outflux of money per unit time, e.g., as the annual income, so their dimensionality is [dollar/time].

Path dependent differentials (δ M) like income and costs are measured in dollar/ cycle time. The length of the cycle time depends on the contract of the partners. Path independent differentials (d K) like capital are measured in dollar, independent of the cycle time. Within one cycle all differentials are given as money, in US $, € etc.

The author must state explicitly what are the dimensionalities of all variables introduced in the paper. The economics subject is unfamiliar to physicists, so the dimensionalities must be spelled out explicitly.

The dimensionalities have been stated for all variables.

4. In the Section around Eq. (9N), the author declares that, in contrast to neoclassical theory, money is not conserved in a closed cycle. However, he does not identify what is the source of this influx of money. In any economic transaction, the money received by one agent (considered as income) is equal to money paid by another agent (considered as spending), so the totals of Y and C over the whole system exactly cancel each other in Eq. (9N), and the total money is conserved in the system. These issues are discussed in detail in the recent review paper arXiv:0905.1518. The analogy with the blood system makes a lot of sense. If the author claims that it is wrong, he needs to identify the source and the mechanism of blood infusion.

The bloodsystem is discussed in detail. Blood / molecules in a body / heat pump is conserved. But the energy carried by the blood in a body, the energy carried by the molecules in a heat pump is not conserved. Food / oil is needed as external resource to keep the body /motor running.
The paper *arXiv:0905.1518* by V. M. Yakovenko and J. B. Rosser has been cited.
Victor Yakovenko as a physicist in thermodynamics agrees with my view on economics.
J. B. Rosser as an economist has given a positive report on my approach to economics:
http://cob.jmu.edu/rosserjb/Econophysics%20and%20Sociophysics.%20book%20review.doc

This is a very important and central conceptual issue. There are two circuits in the economy. One is the circuit of physical goods, and it is governed by physical laws (i.e., technology, energy consumption, etc.). Physical goods are inherently not conserved: e.g., one can plant a seed, grow a tree, and start producing apples every year, which will be eaten or rotten in the end. Another cycle is monetary. It is governed by artificial, but strict rules invented and imposed by human society. Central government has the monopoly for issuing money, and violators (trying to print counterfeit money) are persecuted. So, by law, agents can only receive and to give money, but are not allow to manufacture money, i.e., to grow dollar bills on a money tree. Thus, the only possible source for influx of money into the system is the central government, and it has nothing to do with the material (physical) production in the system.

In a growing economy the government has to print additional money according to growth. The additional money is owned by the government, it is not yet in circulation. A mining company producing gold may sell this gold to the government. Now the additional money has gone into the monetary cycle corresponding to the productivity of the (mining) company.

This should be particularly obvious now, under the current US administration. Traditional economics theory tends to ignore the role of government and tries to convince people somehow that growth of money can be generated within the system. However, the recent economic crisis painfully shows how wrong is this common misconception.

Another way of generating money is borrowing, e.g., by creating negative debt and virtual money. However, as the author says and the current crisis shows, the process of borrowing cannot go forever, and sooner or later the system goes bankrupt and has to be bailed out by the government. To simplify a theoretical study and to ensure stability of the studied system, it is reasonable to consider a system where debt is not permitted. This seems to be the direction taken in the refereed paper regarding debt.

This is true and has been discussed in the paper.

5. In Eq. (14), the author introduces an unknown variable u and unknown function W(u). Introducing an unknown variable undermines that case presented by the author. He claims that the traditional one-dimensional calculus is wrong and should be replaced by two-dimensional calculus. Yet, he cannot explain the meaning of the second variable u. This does not help to make a convincing case for his theory.

This is true, the function W(u) is not very convincing and has been omitted, since it is not needed in the further course of the paper.

Also, the choice of notation is poor. There are variables W and W-bar. The latter represents wages. Are they related anyhow? There is a reference to nonexistent Eq. (3N) in the paragraph after Eq. (19N).

These notations and parameters are now left out.

There are more open questions and imperfections in the current version of the paper, but if the author can address the issues raised above, that would already be a significant progress toward improving the paper.

I thank all referees for their involvement in my paper.