Introduction to Macro-Econophysics and Finance

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Abstract

Closed integrals in physics lead to equations for sources and vortices in fluid mechanics, electrodynamics and thermodynamics. In economics, the Stokes integral of economic circuits leads to new fundamental equations of macro-econophysics. These equations differ significantly from the laws of neoclassical theory. Entropy of markets replaces of the economic Cobb Douglas function and leads to stochastic processes and micro-econophysics of financial markets.

Introduction

Econophysics is the exchange of methods between natural and socio-economic sciences. The term “econophysics” was introduced by E. Stanley in 1995 for financial markets [1]. A good overview has been given by Yakovenko and Rosser [2]. The present paper focuses on the interaction of thermodynamics and macro-economics [3], starting from the question:
“Why are some economic functions like capital predictable and other functions like income and profits are not?” Economists call these functions “putty”, as they will harden only at the end. In contrast “clay” functions are solid in the beginning and at the end [4].

The answer to this problem lies in the structure of economics as a two dimensional theory, which depends on two parameters, capital and labor. This is similar to thermodynamics, which also depends on two variables, temperature and pressure. Calculus in two dimensions offers two different types of integrals: Riemann integrals of exact differential forms like entropy (d S) are path independent, predictable, clay, conservative. Stokes integrals of not exact forms like heat (δQ) are path dependent, unpredictable, putty, non conservative.

Two dimensional calculus leads to a severe problem for neoclassical theory, which is based on one dimensional calculus and the Solow model [5]: Y = F (K, N). Income (Y) is determined by a clay production output function (F), depending on capital (K) and labor (N). But how can a putty income be equal to a clay production function? It is the idea of this paper to solve this contradiction in neoclassical economics by introducing macro-econophysics as a new two dimensional theory of economics and finance.

Production circuits

The French economists and physician Françoise Quesnay (1694 – 1774) has based the natural production circuit on the closed blood stream: In fig. 1 work is transferred from households to agriculture and consumption goods are brought back from agriculture to households. Consumption goods (produce) are the rewards of work or labor input.

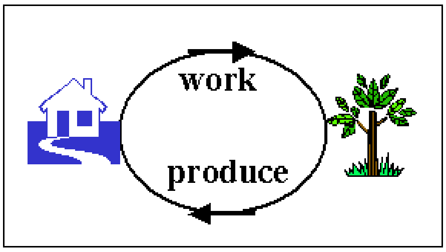

Fig. 1. Quesnay´s model of a simple production circuit of a natural economy. Work of laborers is transferred from households to agriculture. In return produce is brought as a reward from agriculture to households.

The production circuit is an important step beyond linear production models. Work and consumption goods in the production circuit may both be measured in energy units [Q], in Joule, kWh or calories.

According to Irving Fisher [6] modern production may be characterized by two equivalent circuits, the production and the monetary circuit, fig.2.

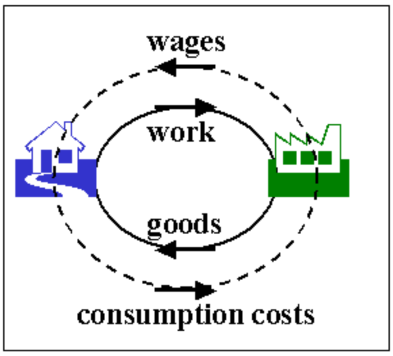

Fig. 2. The model of modern economies of capital (K) and labor (N) contains two putty economic circuits or Stokes integrals: in the production circuit (solid line) households send labor to industry and in return industry sends goods to households. In the monetary circuit (dashed line) industry pays wages to households and households pay consumption costs to industry. The monetary circuit measures the production circuit.

In a modern production circuit laborers from (N) households go to work in industries, the productive capital (K) of modern society, and consumption goods from industry are sent to households. But consumption goods are not the reward for labor like in natural production. In a second, monetary circuit industry pays wages for labor, and households pay consumption costs for goods to industry.

Neoclassical economics interprets the monetary circuit, the balance of income and costs as a closed flux cycle of money:

$$Y_H - C_H = S_H \tag{1}$$

Wages for industrial workers, e.g. $Y_H = 100$ € per day, flow from industry to households. Consumption costs , e.g. $C_H = 90$ € per day, flow back from households to industry. The surplus, $S_H = 10$ € per day, flows to a bank and back to industry as investment.

However, the flux interpretation of monetary circuits cannot be correct: industry spends 100 € per day per laborer and earns 90 € per day from consumer products. Industry then has to borrow 10 € per day from the bank in order to pay 100 € again to each laborer the next day.

No industrial manager will follow this idea. Instead, if labor costs are at 100 € per day, industry will expect to earn at least 120 € from consumer products to make a profit of 20 € per day and per laborer. But these 120 € cannot come from households, if they only make 100 € per day.

For a correct interpretation of monetary circuits economic theory has to be expanded to two dimensions including Riemann and Stokes integrals.
A simple model may explain the difference between neoclassical flux cycles or Riemann integrals and monetary circuits as Stokes integrals:

1. In neoclassical view the economic cycle is a Riemann integral, and works like a closed traffic circle. Cars may enter or leave the traffic circle by any of the incoming roads. The flux of cars is always in one level.
2. In econophysics the economic cycle is a Stokes integral, and works like a parking house. Cars drive in circles to find a parking lot. But each time the car makes another cycle it has reached a higher level.

In the same way production in economic circuits leads to a higher income level after each cycle: The farmer earns his crop at the end of the year, the worker gets his wages at the end of the week, the investor gains his profit only after ending the financial investment. As a result we must leave the one dimensional world of neoclassical models and turn to calculus in two dimensions.

The first law of economics

Production and monetary circuits in fig. 2 are now discussed in terms of two dimensional calculus: The monetary circuit (δM) measures production (δP),

$$\oint \delta M = -\oint \delta P \neq 0 \qquad (2).$$

The negative sign of the Stokes integral indicates the opposite direction of the circuits in fig. 2. Monetary and production circuits are examples of Stokes integrals in economics. The equivalence of monetary and productive circuits in Eq.(2) may also be expressed by differential forms:

$$\delta M = d K - \delta P \qquad (3).$$

The two not exact differentials (δ M) and (δ P) differ by an exact differential (d K), the closed (Riemann) integral of an exact differential is always zero. The closed Stokes integral of Eq.(3) leads to Eq.(2) again. The form (d K) has the same dimension of money like (δ M) and (δ P) and represents capital. Eq.(3) tells us: Profits (δ M) depend on capital (d K) and labor (δ P). This result is well known. But there is more information in Eq.(3): Profits and labor are always putty, capital is always clay. The negative sign of production in Eq.(3) indicates that labor has to be invested in order to make profits. Unlike in neoclassical theory labor is not just labor force (N), but is the actual work or technology performed by the laborers. Eq.(3) is the basic natural law of economics, it is the balance of all economic systems.

Eq.(3) may be compared to the first law of thermodynamics of heat (δQ), energy (d E) and work (δW): $\delta Q = d E - \delta W$. Accordingly, we may call Eq.(3) the first law of economics.

The second law of economics

According to the laws of calculus a not exact differential form (δ M) may be turned into an exact differential form (d F) by an integrating factor (λ):

$$d F = \delta M / \lambda \qquad (4).$$

The clay function F is a system function and is called production function in economic systems. The integrating factor (λ) may be interpreted as a mean capital level of the system: in markets (λ) will be a common price level of a commodity, in societies (λ) will be the mean standard of living.

Eq.(4) corresponds to the second law of thermodynamics, $d S = \delta Q / T$ and may be called second law of economics. The production function (F) corresponds to the entropy function (S) and the standard of living (λ) of an economy to the mean energy level or temperature (T) in physical systems.

The monetary circuit

Eq.(4) may be solved for δ M and be written as

$$\delta M = \lambda \, dF \qquad (5).$$

The closed Stokes integral of money (δM) may be split into two parts,

$$\oint \delta M = \int_A^B \delta M + \int_B^A \delta M = Y_u - C_u = S_u \qquad (6\ a).$$

Eq.(6 a) is the two dimensional monetary circuit and corresponds to the neoclassical balance in Eq.(1). Income, costs and surplus are putty functions and depend on the path (u) of integration. Income (Y_u) and costs (C_u) are defined as integrals of δM, where the limits of integration are A (donor, industry) and B (receiver, households),

$$Y_u = \int_{Ind}^{H} \delta M = \int_{Ind}^{H} \lambda dF \qquad (6\ b).$$

$$-C_u = \int_{H}^{Ind} \delta M = \int_{H}^{Ind} \lambda dF \qquad (6\ c).$$

The path dependent Stokes integrals Eq.(6 a, b, c) replace the neoclassical idea of monetary fluxes. In addition we find the solution for the contradiction in the Solow model: Putty income (Y) is not equal to the clay production function (F). Only for constant λ we obtain

$$Y_u = \lambda F \qquad (6\ d).$$

In general the Solow model must be replaced by the second law, Eq.(5).

Entropy and Production

Combining the first and second laws in Eqs.(3) and (4) to

$$\delta P = dK - \lambda \, dF \qquad (7)$$

shows the relationship of production (P), capital (K), standard of living (λ) and entropy (F). Production requires capital and entropy. Entropy is a measure of disorder of a system, (− d F) means reduction of disorder! Eq.(7) tells us: Production means ordering, putting all parts into the right order. This law is valid in all production lines, a mechanics orders the different parts of a car, a policeman orders traffic, a medical doctor helps to keep the body in order, a teacher orders the minds of students. The monetary value of production (P) depends on the standard of living (λ), in countries with low standard of living like China production is cheaper than in countries with high standard of living.

Entropy and the Cobb Douglas Production Function

In neoclassical theory the optimal output is generally represented by the Cobb Douglas production function F_{CD} [7]. In this chapter we will compare the output of binary production systems at constant number (N) of workers a) according to entropy and b) according to the Cobb Douglas function:
Example: A company has a percentage of x skilled and 1 - x unskilled employees. The production output per employee is given by the production function f(x).

a) The entropy function $f(x_1, x_2)$ of a binary system is given by

$$f(x) = -[x \ln(x) + (1-x)\ln(1-x)] \quad (8\text{ a})$$

$$= -[\ln(x)^{x} + \ln(1-x)^{(1-x)}] \quad (8\text{ b})$$

b) the neoclassic production function (f_{NC}) is

$$f_{CD}(x) = x^{\alpha}(1-x)^{1-\alpha} \quad (8\text{ c}).$$

The Cobb Douglas elasticity exponents α is not defined and is often taken between α = 0,5 and α = 0,7 to obtain a best fit.

The production output per employee f (x) has been plotted in fig. 3 for entropy in Eq.(8 a) and for neoclassical theory (Cobb Douglas) in Eq.(8 c) with exponents α = 0.7 and α = 0.5. Apparently the Cobb Douglas function is not the optimal production function. The entropy function f(x) is larger than the neoclassical function f_{CD} by a factor of about 1.4.

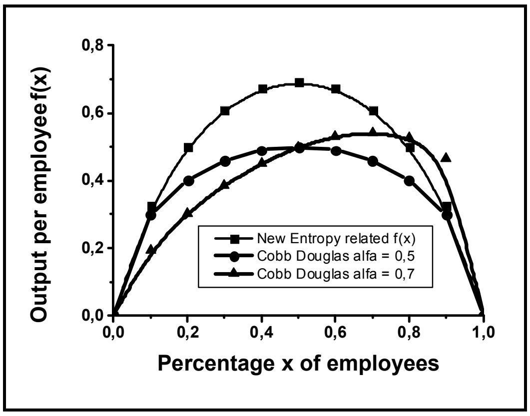

Fig. 3. Production function per employee f (x) for a company with N people at two kinds of jobs. A percentage $x = x_1$ people are skilled and work in job (1) and a portion $x_2 = (1 - x)$ are unskilled working in job (2). The output per employee, Eqn.(8 a) is plotted versus x in the range from 0 to 1. The Cobb Douglas function f_{CD} has been calculated for α = 0,7 and α = 0,5 according to Eq.(8c). The entropy function f(x) is larger than the neoclassical function f_{CD} by a factor of about 1.4.

The Carnot production process

We may now apply the second law to the monetary circuits. In Eq.(6 b, c) the putty properties of income (δ Y) and costs (δ C) have been transferred to a flexible value of λ, as the production function (d F) is clay. The flexible value of λ in a closed path may be given by: $\lambda = \lambda_2$ for one path (income) and $\lambda = \lambda_1$ for the way back (costs):

$$\delta Y \;=\; +\,\lambda_2\;dF \qquad (9\text{ a}).$$

$$\delta C \;=\; -\,\lambda_1\;dF \qquad (9\text{ b}).$$

The closed economic circuit is called Carnot process and may be applied to all economic systems. It corresponds to thermodynamic systems like motors, generators, refrigerators.
Fig. 4 explains the Carnot mechanism of profits in the λ – F plane.

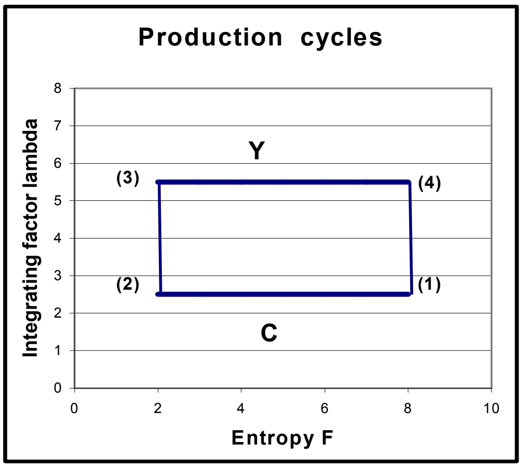

Fig. 4: Carnot cycle of production and trade in the λ – F plane according to the Stokes integral Eqs.(9 a, b) corresponds to financial and production cycles in fig. 2. The calculation is explained in the text.

1. Production cycle at a farm:
1 → 2: Produce is collected by workers at low price level (λ_1).
2 → 3: Produce is brought from the fields (λ_1) to the market (λ_2).
3 → 4: Produce is distributed to customers at higher price level (λ_2).
4 → 1: The garbage is brought back to the fields as fertilizer, (λ_2)→ (λ_1).

2. Monetary cycle:
4 → 3: Income (Y) is collected from customers at high price level(λ_2).
3 → 2: The money is brought from the market (λ_2) to the field (λ_1).
3 → 4: The costs (-C) are distributed to workers at low wage level (λ_1).
4 → 1: The workers buy commodities at the market , (λ_1)→ (λ_2).

The Carnot process also applies to international trade, to manufacturing and trading with countries of lower standard of living (λ_1) like China and selling these commodities in countries with higher standard of living (λ_2) like USA. $\Delta\lambda$ is the change in standard of living bringing commodities from China to the US market. The area ($S_\lambda = \Delta\lambda\ \Delta F$) in fig. 4 is the common surplus of Chinese and US trade.

The Carnot production process is valid for all economic systems like homes, farms, companies, production plants, banks, countries, economies. Production plants and motors even use the same fuel: oil!

The Carnot process always creates two different levels: In economic production it is capital and labor, in banks we have savers and investors, in markets we find buyers and sellers, in societies we have rich and poor! In thermodynamic systems like motors, generators, heat pumps and refrigerators it is hot and cold.

The relative difference ($\Delta\lambda / \lambda$) is called efficiency of the Carnot process,

$$r = \Delta \lambda / \lambda \qquad (10).$$

The higher the difference of the levels ($\Delta\lambda$) the better is the efficiency. A constant level (λ) at both sides of the Carnot process is the end of economic growth, the motor stops. Zero growth may look like a good option for the world economy and population. But a constant level (λ) is not equivalent to an even distribution of resources. The worldwide inequality of income leads to a permanent struggle for economic growth.

Financial markets

Profits in financial markets are again determined by the first law, Eq.(3). Putty profits (δM) may be obtained from capital ($d K$) and production (δP). But investing only in clay capital ($d K$) will lead to zero output,

$$\oint dK = 0 \qquad (11).$$

The Riemann integral Eq.(11) indicates that only investment in putty production (δP) will lead to non zero output. Capital alone cannot create capital. This may be demonstrated by investments in long and short term (US) stocks in figs. 5 and 6.

Long term investment e. g. in (US) stocks corresponds to investment in production (δP) and has shown more than 7 % growth per annum between 1940 and 2000 in fig. 5.

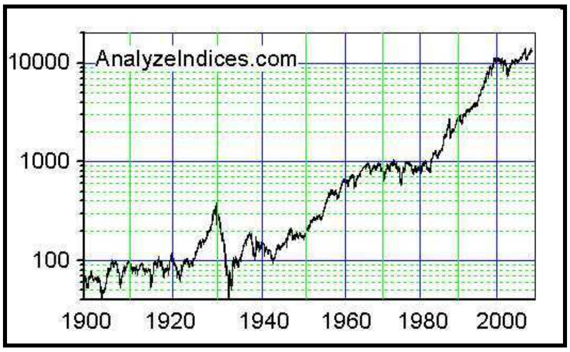

Fig. 5. A long time investment in the (US) stock market corresponds to an investment in putty production (δ P) and shows positive mean returns [8].

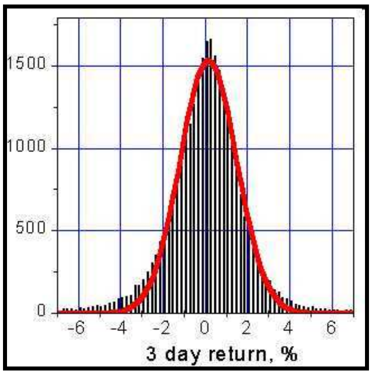

Fig. 6. A short time investment in the stock market corresponds to an investment in capital (d K) and shows symmetric probability of gains and losses [8].

Short time investment corresponds to capital investment (d K). This may seem more appealing, as much higher returns are possible, like between 1920 and 1929. But these high growth bubbles finally burst like in the depressions of 1929 and 2009.

Fig. 6 shows the nearly even distribution of gains and losses of 3 day returns by a bell shaped function with fat tails. Capital investments and gambling do not contribute to growth, they only redistribute wealth. The short term stock market is a legal casino. The only permanent winners are the banks, which collect transfer fees, whether the player wins or loses. In contrast to savings banks investment banks have to invest in financial markets and they will become risky players as well. In many bank strategies debts are paid by new debt. This corresponds to doubling the stake after a loss in the roulette game. The strategy works until probability generates losses that surpass all reserves. At this point the game ends. In the real world states have to rescue their banks to keep the game going.

Conclusion

The economic and financial results above have been obtained from equilibrium thermodynamics. This theory so far has been applied to natural sciences, but, apparently, it may also be successfully applied to social sciences. The results so far indicate that many ideas of neoclassical theory, like the monetary flux of financial circuits, the Solow model, the Cobb Douglas function, and neoclassical growth models have to be replaced by the laws of econophysics based on two dimensional calculus. In addition we may have to look into non equilibrium effects of economics, but here again thermodynamics may be an experienced guide. At present many researchers in the field of econophysics are engaged to establish economic theories that are based on mathematics, natural science and economic experience, which will enable us to cope with present and future productive and financial challenges.

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