Putty and Clay -
Calculus and Neoclassical Theory

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Abstract

Calculus in two dimensions leads to two different types of integrals: Riemann integrals of exact differential forms (df) are path independent, fixed, "clay". Stokes integrals of not exact forms (δf) are path dependent, flexible, "putty". Productive and financial cycles may be represented by Stokes integrals. The resulting differential equations may be regarded as the basic laws of economics: The first law: $\delta Y = d K - \delta P$ relates income (δY) to capital ($d K$) and production (δP). The second law, $\delta Y = \lambda d F$, replaces the Solow model: $Y \neq F(K, N)$. Putty cannot be equal to clay, putty income (δY) is only proportional to the clay production function (dF). The function (F) may be interpreted as the entropy of the economic system and replaces the Cobb Douglas function of neoclassical theory.

Introduction

Econophysics is a relatively new field on the exchange of methods between natural and socio-economic sciences [1-3]. A recent overview has been given by Yakovenko and Rosser [4]. The present paper is focussed on the mathematical background of putty and clay functions. The terms putty and clay have been introduced to economics by Johansen [5] in 1959, and have been discussed in the literature [6, 7]. Putty functions are unpredictable, they are flexible "ex ante" and fixed "ex post". Examples are company profits or income (Y): in the beginning of the year (ex ante) profits are flexible, they may be estimated, but not predicted. At the end of the year (ex post) profits or income are fixed. Clay functions are predictable, they are fixed "ex ante" and "ex post". One example is the production function F(K,N), which may be regarded as a production recipe. Companies need to calculate the fixed production output (ex ante), before they invest in capital

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(K) and labor (N). This fixed value must be the same after production (ex post). Production and economic growth are generally treated by neoclassical theory [8 – 10]. In the present paper production and economic growth are based on two dimensional calculus as they depend on the two parameters or production factors, capital (K) and labor (N). Section 1 introduces Riemann and Stokes integrals and the notations putty and clay in two dimensional calculus. In section 2 monetary and production circuits are shown to be examples of putty Stokes integrals. The resulting differential laws are presented in section 3 as the first and second law of macro-economics. In section 4 entropy is introduced as the new production function of economic systems. Section 5 compares the entropy based Lagrange function of econophysics to the Lagrange function used in economic applications. In section 6 we come to the conclusion that entropy as the new production function replaces the Cobb Douglas function and questions many results that have been obtained so far from the Solow model of neoclassical theory.

1 Putty and clay in two dimensional calculus

1.1 Exact differential forms

In two dimensional space (x, y) the exact differential form d f of a function f(x, y) is generally marked by a “d” and is given by [11,12]

$$
\begin{aligned}
d f (x, y) &= (\partial f / \partial x)\, d x + (\partial f / \partial y)\, d y \\
&= a(x, y)\, d x + b(x, y)\, d y \qquad (1.1)
\end{aligned}
$$

The mixed second derivatives of “d f” will always be equal,

$$\partial^2 f / \partial y \partial x = \partial b / \partial x = \partial a / \partial y = \partial^2 f / \partial x \partial y \qquad (1.2).$$

1.2. Riemann integral of exact or clay differential forms

Integrals of exact differential forms are called Riemann integrals, they depend on the integral limits A and B, but not on the path (u) of integration,

$$f(x, y) = \int_u df(x, y) \qquad (1.3).$$

d f may be called a “clay” differential form, it may be integrated without knowing the path of integration in advance, since the stem function f (x, y) is independent of the path. The closed Riemann integral along a closed path in the x – y plane will always be zero,

$$\oint df(x, y) = 0 \qquad (1.4).$$

1.3. Not exact differential forms

Differential forms in two dimensions are generally not exact. A not exact differential form δ g,

$$\delta g\,(x, y) = a\,(x, y)\,d x + b\,(x, y)\,d y \qquad (1.5)$$

is marked by a "δ". The mixed derivatives of δ g will not be equal,

$$\partial b / \partial x \neq \partial a / \partial y \neq \partial^2 f / \partial x \partial y \qquad (1.6).$$

1.4. Stokes integral of not exact or putty differential forms

Integrals of not exact forms in two dimensions are called Stokes integrals, they depend on the integral limits A , B and on the path (u) of integration,

$$g_u(x, y) = \int_u \delta g(x, y) \qquad (1.7).$$

The differential δ g may be called a "putty" differential form, it may only be integrated, after the specific path is known. A stem function g (x, y) does not exist, only functions g_u (x, y), which are different for each path (u).

The closed integral of δ g along a closed line in the x – y plane is never zero,

$$\oint \delta g(x, y) \neq 0 \qquad (1.8).$$

The closed integral may be divided into two open integrals, from A = (x_0, y_0) to B = (x_1, y_1) and back from B to A:

$$\oint \delta g(x, y) = \int_{x_0, y_0}^{x_1, y_1} \delta g + \int_{x_1, y_1}^{x_0, y_0} \delta g = \int_{x_0, y_0}^{x_1, y_1} \delta g - \int_{x_0, y_0}^{x_1, y_1} \delta g \neq 0 \qquad (1.9).$$

The integrals along a closed line in the x – y plane will not cancel. They have the same limits, but the path (u) is different.

1.5. Integrating factor

A not exact putty form in two dimensions, δ g (x, y) may be turned into an exact clay form d f (x, y) by an integrating factor λ,

$$d f(x, y) = (1 / \lambda)\, \delta g(x, y) \qquad (1.10)$$

In two dimensions the function f (x, y) and the integrating factor λ (x, y) always exist.

In the following sections calculus of putty and clay differential forms will now be applied to economics.

2 Economic Circuits

2.1 The natural production circuit by Quesnay

One of the first economists, the French medical doctor Françoise Quesnay (1694 – 1774) was inspired by the human blood circuit and looked at natural production not as a continuous process, but as a production circuit: Every day workers from households go to work in the fields, the capital of the village, and bring back home consumption goods from the fields.

Consumption goods are the reward for labor in the fields. The labor invested in the fields is directly related to the output of produce: a hard worker will pick many apples, less hard work leads to fewer apples. Work and consumption goods are part of the same production circuit and can be measured in units of energy per circuit, in Joules or kWh. The circuit or number of cycles is dimensionless.

2.2 Modern production and monetary circuits

Macro-econophysics is based on Quesnay´s cyclic approach and regards the closed production circuits as Stokes integrals of work or production (δ P),

$$\oint \delta P = \Delta P_u \neq 0 \qquad (2.1).$$

The closed cycle, the Stokes integral of production is an “ex post” process, as the production output ΔP_u is not zero.

In modern production the output is not the reward of labor input. There is a second cycle, a monetary or financial circuit. Industry pays wages (Y_H) for labor to households, and households pay consumption costs (C_H) for goods to industry. A second monetary or financial circuit (δ Y) measures the

production circuit (δ P) – not in units of energy per cycle, but in monetary units per cycle, like US $, €, British £ or Japanese ¥.
1. The equivalence of monetary circuit (δ Y) and production circuit (δ P) in may be expressed by the closed Stokes integrals

$$\oint \delta Y = -\oint \delta P \neq 0 \qquad (2.2).$$

The negative sign of the Stokes integral of (δ P) indicates the opposite direction of the circuits (δ Y) and (δ P).
2. The equivalence of production and monetary circuits corresponds to equilibrium of supplies of producers (δ P) and demand of buyers, who pay the agreed amount of money (δ Y).
3. The cycle length is generally part of a contract between the parties.
4. Surplus (S_u) may be positive or negative. Positive surplus or profits are due to productive forces, negative surplus or losses are due to frictional forces.
5. The equivalence of monetary (δ Y) and production circuit (δ P) in Eq.(2.2) are the basis for all macro econophysical calculations.

2.3 Modern monetary circuits

1. The value of the Stokes integral of the monetary circuit is called profit or surplus (S_u), the index “u” stands for a specific production path or process.

$$\oint \delta Y = \pm S_u \qquad (2.3).$$

2. The closed Stokes integral (2.3) may be split into two parts,

$$\oint \delta Y = \int_A^B \delta Y + \int_B^A \delta Y = Y_u - C_u = S_u \qquad (2.4).$$

Income and costs are defined by open integrals, where the limits of integration are A (donor) and B (receiver).

$$Y_u = \int_A^B \delta Y \qquad (2.5).$$

$$-C_u = \int_B^A \delta Y \qquad (2.6).$$

Income, costs and surplus are “putty” functions, they are path dependent and cannot be calculated until the money is paid.

3 Differential laws of economics

3.1 First law of economics

1. The equivalence of monetary (δ Y) and productive (δ P) circuits in Eq.(2.2) is expressed by Stokes integrals. The law of equivalence may also be given in differential forms:

$$\delta Y = d K - \delta P \qquad (3.1).$$

Eq.(3.1) is the balance of economic systems and states a well known fact: *Profits (δY) depend on capital (d K) and labor or work (δP).*
Income and profit (δ Y) are generated by production (δ P). The negative sign of (δ P) indicates that labor has to be invested in order to make profits. Eq.(3.1) is a basic differential law of economics.
2. In addition we obtain an “ex ante” differential form (d K), as the closed integral of an exact form (d K) is zero, Eq.(1.4).

Income (δ Y) and labor (δ P) are measured in monetary units. This must also be true for (d K): the function (K) represents capital, the only monetary variable, that has not been discussed in econophysics, so far. The exact differential form of capital (d K) drops out of the Stokes integral (2.2), this means: Income is generated by labor (δ P), capital cannot generate capital: *“It was not by gold or by silver, but by labour, that all the wealth of the world was originally purchased.*” (Adam Smith, wealth of nations I.6.11).
3. Eq. (3.1) may be compared to the first law of thermodynamics of heat (Q), energy (E) and work (W): $\delta Q = d E - \delta W$, accordingly, we may call it the first law of economics.

3.2 Second law of economics

A not exact “putty” differential form (δ Y) may be turned into an exact “clay” differential form (d F) by an integrating factor (λ), Eq.(1.10):

$$d F = \delta Y / \lambda \qquad (3.2).$$

1. The function F may be called production function. In all economic systems a clay production function (F) will exist “ex ante”.
2. The integrating factor (λ) exists in all economic systems, in production, in markets and finance.
3. Eq.(3.2) corresponds to the second law of thermodynamics, $d S = \delta Q / T$ and may be called second law of economics. The production function (F) corresponds to the entropy function (S), the price level (λ) of a market

corresponds to the mean energy level or temperature (T) in physical systems.
4. Classical economic theory assumes the existence of an "ex ante" production function (F). This assumption is confirmed by calculus in Eq.(3.2). We may solve Eq.(3.2) for δY and obtain

$$\delta Y = \lambda \, dF \qquad (3.3).$$

5. Eq.(3.2) relates "putty" (δY) income to the "clay" production function (F) by an integrating factor (λ). This result is in contrast to the Solow model of neoclassical theory: $Y \neq F(K, N)$, putty income cannot be equal to the clay production function. The Solow model cannot be correct!

3.2.1 Production function F

The production function F of the second law of economics exists for every closed economic system. In a farmers market the production function is given by the amount of wheat, corn, produce, fruits that is offered. In a company the production function is related to the number of people in different jobs, the number of workers, engineers, clerks, drivers, secretaries, managers etc. in the company. In economies the production function is determined by the number of different factories, companies, business firms of a country. As the production function depends on the numbers (N) of elements in the different economic systems, the function F (N) will be dimensionless.

3.2.2 Integrating factor λ

The integrating factor (λ) corresponds to temperature (T) in physical systems. Temperature is the mean kinetic energy per particle of a closed physical system. In economics efficient markets at equilibrium will lead to zero arbitrage, this means a common price level (λ) per item will evolve for the same product. In societies (λ) is the living standard of a population, in economies (λ) is the GDP per capita. Like temperature (T) in physics we expect (λ) to one of the leading functions in economics. This will be discussed in more detail, below.

4 Entropy as production function

4.1 Entropy and probability

The production function F is given by entropy, like in physical systems. This has already been pointed out by Roegen [13] and Jaynes [14]. Entropy depends on the number of possibilities Ω to place N elements into k different categories. The production function F of a company is the number Ω of possibilities to place N people in k different professions.

$$F = \ln \Omega \qquad (4.1).$$

The number of possibilities Ω is the number of possible combinations,

$$\Omega = N! / \Pi_k N_k ! \qquad (4.2).$$

Applying the Sterling approximation $\ln (N !) = N \ln N - N$, we obtain for the entropy production function

$$F = \ln \Omega = N \ln N - \Sigma_k N_k \ln N_k \qquad (4.3).$$

The entropy production function F depends in a logarithmic way on the number N_k of elements in each of the k categories, like the number N_k of employees in each of the k professional groups in accompany or the number N_k of fruits for each fruit k at a fruit market.The number N represents the total number of items, the total number of employees in a company, the total number of fruits in a fruit market. The function F is uniquely defined in each economic system (company, market) and has no adjustable parameters.

4.2 Relative binary entropic production function f (N_1, N_2)

The number of people N_k in each profession may be substituted by the relative number $x_k = N_k / N$. Dividing Eq.(4.3) by the number N of employees we get the entropic production function per capita f = F / N:

$$f = F / N = - \Sigma_k x_k \ln x_k \qquad (4.4).$$

Economic applications often consider only the competition between two different groups, k = 2. In a company a percentage x_1 of the people work in the office and a percentage x_2 in production. This leads to

$$f (x_1, x_2) = - (x_1 \ln x_1 + x_2 \ln x_2) \qquad (4.5).$$

f is the entropic production function or output per capita.
With $x_1 + x_2 = 1$ and replacing the variable x_1 by x ($x \equiv x_1$) we obtain

$$f (x) = - x \ln x + (1 - x) \ln (1 - x) \qquad (4.6)$$

In binary systems the production function per capita f(x) depends only on one parameter x, e. g. the size of the larger group. Eq.(4.6) will be applied to problems with a constant number N of employees in the next paragraph.

4.3 Entropy and Cobb Douglas production function

In a company with a given number of employees a percentage x of the people works in the office and (1-x) of the people work in production. The relative production function of neoclassical theory is given by the Cobb Douglas function per capita,

$$f_{CD}(x) = x^{\alpha}(1-x)^{1-\alpha} \tag{4.7}$$

The output per person f_{CD} depends now on one parameter x, the percentage of people in the larger group, and the uncertain value of elasticity α ! The function f (x) in Eq.(4.6) and the Cobb Douglas function f_{CD} have been compared in fig. 1 for a constant number of people in two different jobs. The Cobb Douglas exponents α are assumed to be $\alpha = 0,7$ and $\alpha = 0,5$.

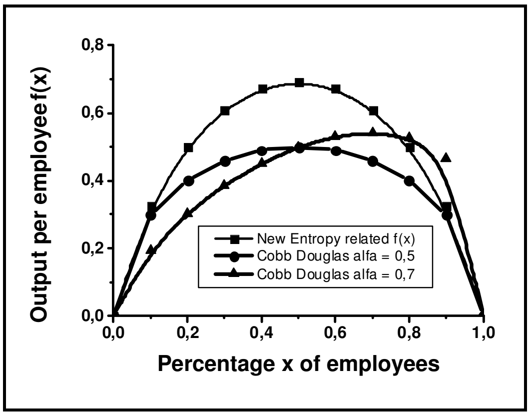

Fig. 1. Production function per employee f (x) for a company with N people at two kinds of jobs. A percentage $x = x_1$ people work in job (1) and x_2 in job (2). The output per employee, eq.(4.6) is plotted versus x in the range from 0 to 1. The Cobb Douglas function per capita, f_{CD} in eq.(4.7) has been calculated for $\alpha = 0,7$ and $\alpha = 0,5$. The value of elasticity α remains uncertain.

The maximum output per employee of the entropic production function f is found at $x_{max} = 0,5$. The maximum output is $f(x = ½) = \ln 2 = 0,693$.
The maximum output of the Cobb Douglas function f_{CD} at $x_{max} = \alpha$ remains uncertain like α. For $\alpha = 0,5$ we obtain $f_{CD}(x = ½) = 0,500$.

The entropy production function f(x) is nearly always larger than the Cobb Douglas function f_{CD} by a factor of A = 1,4. Apparently the Cobb Douglas function requires a factor A = 1,4 as an approximation to the real entropy production function, Eq.(4.6).

5 Lagrange function

5.1 Variation of the Lagrange function

The first and second laws (3.1) and (3.2) may be combined eliminating δY,

$$\delta P = d K - \delta Y = d K - \lambda d F \quad (5.1).$$

By subtracting and adding $F d \lambda$ we do not change the value of δP.
With $d(\lambda F) = \lambda d F + F d \lambda$ we obtain:

$$\delta P = d(K - \lambda F) + F d \lambda \quad (5.2).$$

$$\delta P = d L + F d \lambda \quad (5.3).$$

$$L = (K - \lambda F) \quad (5.4).$$

The function L is called Lagrange function. This function L exists in all economic systems. This function has an important economic feature. Markets have to optimize the amount of specific commodities and also the price. Companies have to determine the number of people working at each job, and also the salary of the people. The mathematical answer to these problem is given by the Lagrange principle:

$$L = K - \lambda F \rightarrow \text{minimum!} \quad (5.5)$$

L is the Lagrange function that has to be minimized. F is the production function, K the capital or budget restriction. λ is the integrating factor and is also called Lagrange parameter. Eq.(5.5) indicates that in optimal markets costs K are always at minimum and the production function F is always at maximum. Examples are given in the next paragraphs.

5.2 Entropy and budget restrictions

A company has N_1 permanent and N_2 temporary employees. The wages per hour are w_1 for the permanent and w_2 for the temporary staff. The total costs of wages are

$$K = N_1 w_1 + N_2 w_2 \quad (5.6)$$

At constant number (N) of employees the Lagrange function (5.4) may be minimized with respect to N_1 and N_2,

$$L = (N_1 w_1 + N_2 w_2) \quad (5.7)$$
$$- \lambda \{(N_1+N_2) \ln (N_1+N_2) - N_1 \ln N_1 - N_2 \ln N_2\} = \text{min!}$$

At minimum the derivation with respect to N_1 and N_2 will be zero:

$$\partial L / \partial N_1 = \ln (N_1 + N_2) + 1 - N_1 \ln N_1 - 1 - w_1 / \lambda = 0 \quad (5.8)$$

$$\partial L / \partial N_2 = \ln (N_1 + N_2) + 1 - N_2 \ln N_2 - 1 - w_2 / \lambda = 0 \quad (5.9)$$

Introducing the relative numbers of staff x_1 and x_2, the calculated distribution of people in the two jobs follows a Boltzmann distribution,

$$x_1 = N_1 / N = \exp(- w_1 / \lambda) \quad (5.10)$$
$$x_2 = N_2 / N = \exp(- w_2 / \lambda) \quad (5.11)$$

The relative numbers of permanent and temporary staff x_1 and x_2, the Lagrange parameter λ, the mean output per person $f = F / N$ and the mean wages per person may be calculated from the wages w_1 and w_2 without any further assumptions:.

5.2.1 Entropy and budget restrictions of a restaurant

A restaurant has a constant number (N) of employees, N_1 permanent staff and N_2 temporary helpers. The wages per hour are $w_1 = 15$ € for the permanent and $w_2 = 7{,}5$ € for the temporary staff. With the given values of w_1 and w_2 we obtain

$$\begin{aligned} x_1^0 &= \exp(- w_1 / \lambda) &= 0{,}38 \\ x_2^0 &= \exp(- w_2 / \lambda) &= 0{,}62 \\ \lambda &= &15{,}58 \\ f^0 &= &0{,}664 \\ K / N &= &10{,}365 \end{aligned}$$

The relative numbers of permanent and temporary staff x_1 and x_2, the Lagrange parameter λ, the mean output per person $f = F / N$ and the mean wages per person have been calculated from the wages w_1 and w_2 without any further assumptions.

5.2.2. Graphic solution of budget restrictions

Three lines lead to the point (x_{10} ; x_{20}) of optimal production, fig. 2:

(1). The first (straight) line is given by the constant total number N of employees, $x_1 = N_1 / N$ and $x_2 = N_2 / N$ or

$$x_2 = 1 - x_1 \quad (5.12).$$

(2). The second line is the curve of maximal production, it is defined by eqs.(5.8) and (5.9),

$$x_2 = x_1^{w_2 / w_1} \quad (5.13).$$

This curve intersects the first line, eq.(5.12), at to the point (x_{10} ; x_{20}) of optimal production.

(3). The third line, the budget restriction (5.6) may be constructed by

$$x_2 \;=\; x_{20} - (w_1 / w_2) \cdot (x_1 - x_{10}) \qquad (5.14)$$

after the point of optimal production (x_{10} ; x_{20}) has been determined from condition (1) and (2).

(4). A fourth line is given by the iso- production line. However, the entropy production function, eq.(4.6),

$$f^0 \;=\; -\; (x_1 \ln x_1 + x_2 \ln x_2) \;= 0{,}664 \qquad (5.15).$$

is an implicit function and cannot be solved analytically for x_2 at a constant f^0. The iso-production function can only be found by employing a computer program and is not shown in fig. 2. However, as the budget restriction is the tangent of iso-production, this line is not needed for constructing the point of optimal production (x_{10} ; x_{20}).

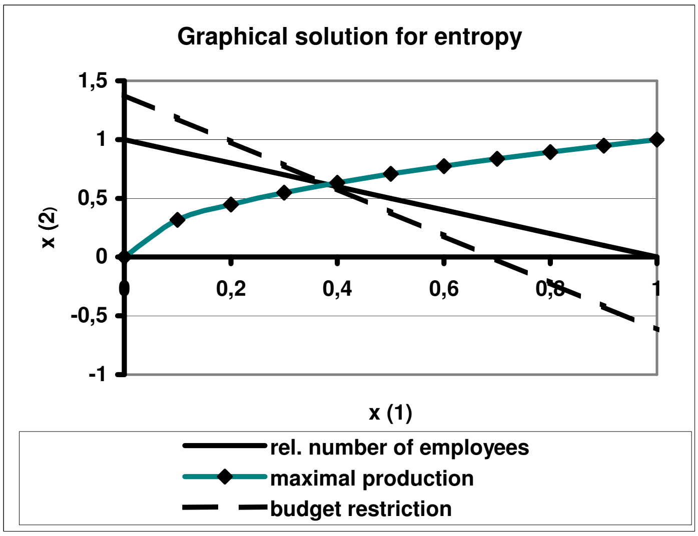

Fig. 2. Graphical solution of optimal production at minimal costs: Three functions lead to the optimal point (x_{10} , x_{20}) = (0,38; 0,62): 1. The relative number of employees, 2. the line of maximal production according to entropy. 3. the budget restriction. 4. the iso- production line at constant entropy is implicit and cannot be shown analytically.

5.3 Budget restrictions according to Cobb Douglas

In neoclassical economics the Cobb Douglas production function F_{CD}

$$F_{CD} \;=\; N_1{}^{\alpha} N_2{}^{1-\alpha} \;=\; N\, x_1{}^{\alpha} x_2{}^{1-\alpha} \qquad (5.16)$$

is applied in the Lagrange function (4.9). In addition to wages E_k employees are rated by an additional parameter α. For an arbitrary value α = 0,7 we obtain

$$x_1 \;=\; \alpha / [\, \alpha / E_1 + (1 - \alpha) / E_2] / E_1 \;=\; 0{,}538$$

$$x_2 = \alpha/[\alpha/E_1 + (1-\alpha)/E_2]/E_2 = 0{,}462$$
$$f^0_{CD} = x_1^{\alpha} x_2^{1-\alpha} = 0{,}5141$$
$$K/N = (w_1 x_1 + w_2 x_2) = 11{,}54$$

5.3.1 Graphic solution of budget restrictions (Cobb Douglas)

The optimal production at minimal costs may also be taken from a graphic solution, fig 3.

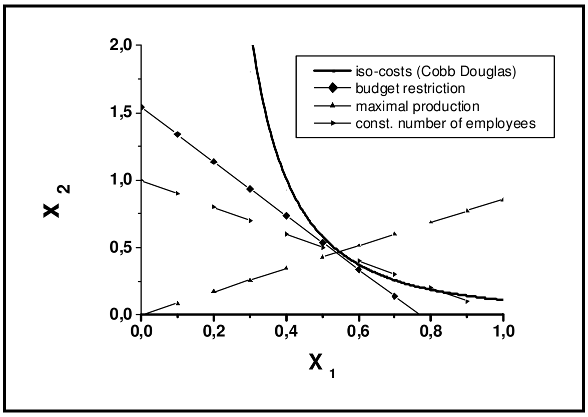

Fig. 3. Graphical solution of optimal production at minimal costs according to Cobb Douglas with $\alpha = 0{,}7$: Four functions lead to the optimal point of production (x_{10} , x_{20}) = (0,538; 0,462): 1. The constant number of total employees, 2. the line of optimal production according to Cobb Douglas, 3. the budget restriction, 4. the iso- production line according to Cobb Douglas. The solutions depend on the choice of the elasticity parameter α.

Again four functions lead to optimal production according to Cobb Douglas:
(1) The constant total number of employees, $x_1 = N_1 / N$ and $x_2 = N_2 / N$,

$$x_2 = 1 - x_1$$

(2) The second line is the line of maximal production according to Cobb Douglas,

$$x_2 = x_1 \, [(1-\alpha)/\alpha] \, (w_1 / w_2)$$

(3) The budget restriction is given by Eq.(5.14),

$$x_2 = x_{20} - (w_1 / w_2) \cdot (x_1 - x_{10})$$

and may be constructed after the point of optimal production (x_{10} ; x_{20}) has been determined from condition (1) and (2).

(4) The iso- production line is found by solving the Cobb Douglas function for x_2 at constant production f^0_{CD} :

$$x_2 = f^0_{CD}{}^{1/(1-\alpha)} \,/\, x_1^{\alpha/(1-\alpha)}$$

but this function is not needed for the constructing of the point of optimal production (x_{10} ; x_{20}). In fig. 3 the four lines meet at the point of optimal production at minimal costs according to Cobb Douglas, which depends on the choice of elasticity parameter $\alpha = 0{,}7$.

5.4. Comparing entropy to the Cobb Douglas function

Comparing the entropy function to the Cobb Douglas function with $\alpha = 0{,}7$ leads to the following optimal parameters:

Parameters	**Entropy**	**Cobb Douglas**
x_{10}	0,38	0,54
x_{20}	0,62	0,46
f^0	0,66	0,51
K / N	10,36	11,54

Table I: Optimal parameters according to
a) entropy and b) the Cobb Douglas function

For all values of α the Cobb Douglas function differs from the entropy data. The optimal point of production (x_1 , x_2) differs from entropy, the mean output f_{CD} is lower and the mean wage costs K / N of the Cobb Douglas function are higher than for the entropy related production function f.

The Cobb Douglas function obviously is not the optimal production function. In addition the results always depend on the arbitrary parameter of elasticity α. This makes neoclassical theory flexible, but inexact! For exact economic calculations neoclassical theory must be replaced by the first and second law based on two dimensional calculus. This new approach leads to new concepts in production and in the theory of finance.

6 Conclusions

The tools of two dimensional calculus lead to a new foundation of production and economic growth. The resulting laws may be regarded as the natural basis of macro-economics. The new laws replace the Solow model of growth in neoclassical theory, the Cobb Douglas production function is replaced by the entropy function.

The Stokes laws of macro-economics are in complement to the principle of entropy maximization [14], which may be regarded as a tool of micro-economics. Beyond this we may apply Stokes integrals and entropy maximization to many specific fields of economics, e. g. financial markets [15] and the financial crisis.

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