Sub-shot-noise phase quadrature measurement of intense light beams

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Received February 9, 2004

We present a setup for performing sub-shot-noise measurements of the phase quadrature of intense pulsed light without the use of a separate local oscillator. A Mach–Zehnder interferometer with an unbalanced arm length is used to detect the fluctuations of the phase quadrature at a single sideband frequency. With this setup, the nonseparability of a pair of quadrature-entangled beams is demonstrated experimentally. © 2004 Optical Society of America

OCIS codes: 270.0270, 270.2500, 270.5570.

For many applications in quantum communication with continuous variables, such as quantum teleportation,[1] entanglement swapping,[2] and quantum cryptography,[3] it is required that one measure the amplitude and the phase quadrature of the electromagnetic field. A homodyne detector[4] is usually applied to perform phase-sensitive measurements by interference of the signal beam and probing of the sidebands with a local oscillator, which is required to be much brighter than the signal beam. However, for intense signal beams, this requirement gives rise to technical difficulties because the high intensities may saturate the detectors. Phase measurements can also be achieved by rotating the bright carrier (internal local oscillator) with respect to the sidebands. Such frequency-dependent phase shifts could be accomplished by reflecting the light of a cavity as a result of multiple beam interference. This technique was used in early quantum optical experiments.[5,6] However, for ultrashort light-pulse trains the requirements for the dispersion properties of the resonator are quite demanding. In this Letter we present an alternative approach in which the fluctuations of the phase quadrature are measured at a certain sideband frequency without the use of any local oscillator or resonator. An interferometric setup reminiscent of that used by Inoue and Yamamoto to determine the longitudinal mode partition noise[7] is shown to be capable of performing quantum-optical measurements of the phase quadrature below the shot-noise level, by introducing a phase shift between the carrier and the sidebands that is due to two-beam interference. Our setup allows for easy switching between the measurement of the phase quadrature and the amplitude quadrature and was used to fully characterize quadrature entanglement of a pair of intense pulsed beams.

The basic setup is a Mach–Zehnder interferometer (see the inset of Fig. 1) with an arm-length difference ΔL that introduces a frequency-dependent phase shift at a certain rf sideband. For frequencies in the megahertz regime, ΔL is much larger than the optical wavelength, typically several meters. To describe the interferometer in terms of quantum theory, we express the input field mode through annihilation operator $\hat{a}(t) = \alpha + \delta\hat{a}(t)$, where α is the classical amplitude (α is assumed to be real) and where $\delta\hat{a}(t)$ contains all classical and quantum-mechanical fluctuations and has zero mean value. This mode is split at a 50/50 beam splitter and hence mixed with vacuum $\delta\hat{v}(t)$. The two emerging modes, denoted $\hat{e}(t)$ and $\hat{f}(t)$, are combined at a second 50/50 beam splitter. The relative optical phase shift between $\hat{e}(t)$ and $\hat{f}(t)$ is denoted φ, the relative delay between the long and the short arms of the interferometer is given by $\tau = \Delta L/c$. Output modes $\hat{c}(t) = 1/\sqrt{2}[\hat{e}(t) + \hat{f}(t)]$ and $\hat{d}(t) = 1/\sqrt{2}[\hat{e}(t) - \hat{f}(t)]$ are then given by

$$\hat{c}(t) = \frac{1}{2}[\alpha + \delta\hat{a}(t) + \delta\hat{v}(t) + \exp(i\varphi)\alpha + \exp(i\varphi)\delta\hat{a}(t-\tau) - \exp(i\varphi)\delta\hat{v}(t-\tau)], \quad (1)$$

$$\hat{d}(t) = \frac{1}{2}[\alpha + \delta\hat{a}(t) + \delta\hat{v}(t) - \exp(i\varphi)\alpha - \exp(i\varphi)\delta\hat{a}(t-\tau) + \exp(i\varphi)\delta\hat{v}(t-\tau)]. \quad (2)$$

For intense states of light the interferometer equations can be simplified by use of a linearization approach.

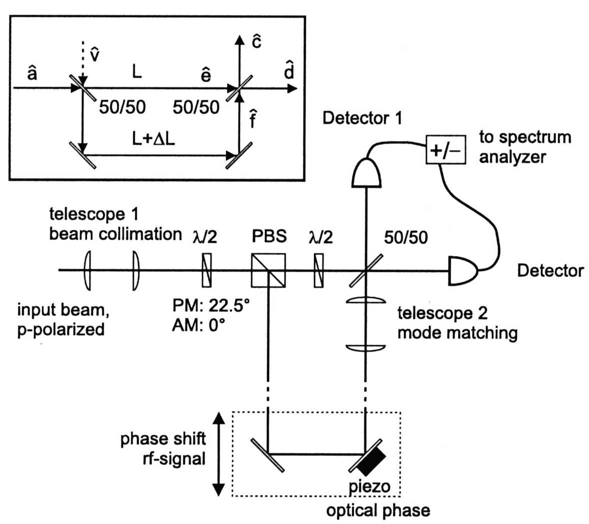

Fig. 1. Experimental setup of the phase-measuring interferometer. The orientation of the first $\lambda/2$-plate determines the type of measurement: amplitude measurement (AM) or phase measurement (PM). PBS, polarizing beam splitter; 50/50, beam splitter.

Photon numbers $\hat{n}_c(t) = \hat{c}^\dagger \hat{c}$ and $\hat{n}_d(t) = \hat{d}^\dagger \hat{d}$ in the two output ports of the interferometer are calculated by keeping the fluctuating contributions up to linear terms. Evaluation of the sum $\hat{n}_c(t) + \hat{n}_d(t)$ and the difference $\hat{n}_c(t) - \hat{n}_d(t)$ of the photocurrents yields

$$\hat{n}_c(t) + \hat{n}_d(t) = \alpha^2 + \frac{1}{2}\alpha[\delta\hat{X}_{a,0}(t) + \delta\hat{X}_{v,0}(t) + \delta\hat{X}_{a,0}(t-\tau) - \delta\hat{X}_{v,0}(t-\tau)], \quad (3)$$

$$\hat{n}_c(t) - \hat{n}_d(t) = \alpha^2 \cos\varphi + \frac{1}{2}\alpha[\delta\hat{X}_{a,-\varphi}(t-\tau) - \delta\hat{X}_{v,-\varphi}(t-\tau) + \delta\hat{X}_{a,\varphi}(t) + \delta\hat{X}_{v,\varphi}(t)], \quad (4)$$

where quadrature component $\delta\hat{X}_{a,\varphi}$ is defined by $\delta\hat{X}_{a,\varphi} = \exp(i\varphi)\delta\hat{a}^\dagger + \exp(-i\varphi)\delta\hat{a}$.

Then, via Fourier transform, the spectral components of the rf fluctuations at sideband frequency Ω are obtained. Since the phase shift of the spectral components is given by $\Omega\tau = \theta$ [note that Fourier transformation $\mathcal{F}$ gives $\mathcal{F}f(t-\tau) = \exp(-i\Omega\tau)\mathcal{F}f(t)$], and the optical phase is adjusted to $\varphi = \pi/2 + 2k\pi$ (k is an integer), the fluctuations of the sum and the difference photocurrents in frequency space read as

$$\delta\hat{n}_c^\Omega + \delta\hat{n}_d^\Omega = \frac{1}{2}\alpha[\delta\hat{X}_{a,0}^\Omega + \exp(-i\theta)\delta\hat{X}_{a,0}^\Omega + \delta\hat{X}_{v,0}^\Omega - \exp(-i\theta)\delta\hat{X}_{v,0}^\Omega], \quad (5)$$

$$\delta\hat{n}_c^\Omega - \delta\hat{n}_d^\Omega = \frac{1}{2}\alpha[\delta\hat{X}_{a,\pi/2}^\Omega + \exp(-i\theta)\delta\hat{X}_{a,-\pi/2}^\Omega + \delta\hat{X}_{v,\pi/2}^\Omega - \exp(-i\theta)\delta\hat{X}_{v,-\pi/2}^\Omega]. \quad (6)$$

For $\theta = \pi$, the sum signal yields $\delta\hat{n}_c^\Omega - \delta\hat{n}_d^\Omega = \alpha\delta\hat{X}_{v,0}^\Omega$ and the difference signal $\delta\hat{n}_c^\Omega - \delta\hat{n}_d^\Omega = \alpha\delta\hat{X}_{a,\pi/2}^\Omega$, which is proportional to spectral component Ω of the phase quadrature of the initial field. The delay must therefore be chosen such that a phase shift of π between the two arms of the interferometer is introduced at measurement frequency $\Omega_m = 2\pi f_m$. Corresponding delay ΔL is then given by $\Delta L = cT/2 = c\pi/\Omega_m = c/2f_m$, where c is the speed of light and T is the period of the rf signal at frequency Ω_m.

The experimental setup of the phase-measuring interferometer is depicted in Fig. 1. It contains a Mach–Zehnder interferometer followed by a balanced detection system using high-efficiency InGaAs photodiodes (Epitaxx ETX 500). The first beam splitter in the interferometer is made of a polarizing beam splitter and a $\lambda/2$ plate. It is therefore possible to switch between phase measurement creating equal intensity in both arms and amplitude measurement, where all light propagates through one arm. The latter situation is equivalent to a balanced detection scheme in which the sum and the difference signals provide the amplitude noise and the shot-noise level, respectively. The variances of the photocurrent fluctuations were recorded with a pair of spectrum analyzers (8590 from HP) at a resolution bandwidth of 300 kHz and a video bandwidth of 30 Hz. The measurement time for each recorded trace as in Fig. 2 was 5 s. The difference of the dc powers of the two detectors served as the error signal and was fed back onto a piezo mirror in the interferometer to stabilize the optical phase.

The quantum source that we characterize with our phase-measuring device produces intense quadrature-entangled light pulses. An optical parametric oscillator pumped by a mode-locked Ti:sapphire laser is used as a light source. It produces pulses of 100 fs at a center wavelength of 1530 nm and at a repetition rate of 82 MHz. The nonlinear Kerr effect experienced by intense pulses in optical fibers[8] is used to generate amplitude-squeezed light pulses employing an asymmetric fiber Sagnac interferometer.[9,10] By use of the linear interference of amplitude squeezed beams, intense entangled light was generated with the fiber-optic setup described by Silberhorn *et al.*[11] To demonstrate sub-shot-noise performance of the interferometer each of the entangled beams was directed into a phase-measuring interferometer. We verified the correlations of the detected photocurrents and compared them with the corresponding shot-noise level.

Working with a pulsed system, one can perform phase measurements only at certain frequencies where interference occurs, as possible delays are governed by the repetition frequency f_{rep} of the laser source, $\Delta L = cnT_{\text{rep}} = cn/f_{\text{rep}}$ (n is an integer number; T_{rep} is the time between two pulses). Possible measurement frequencies are then given by $f_m = f_{\text{rep}}/2n$. In our case, with a repetition rate of 82 MHz, the arm-length difference must be a multiple of 3.66 m, corresponding to the distance between two successive pulses. For a frequency of 20.5 MHz an arm-length difference of 7.32 m is required. To achieve high interference

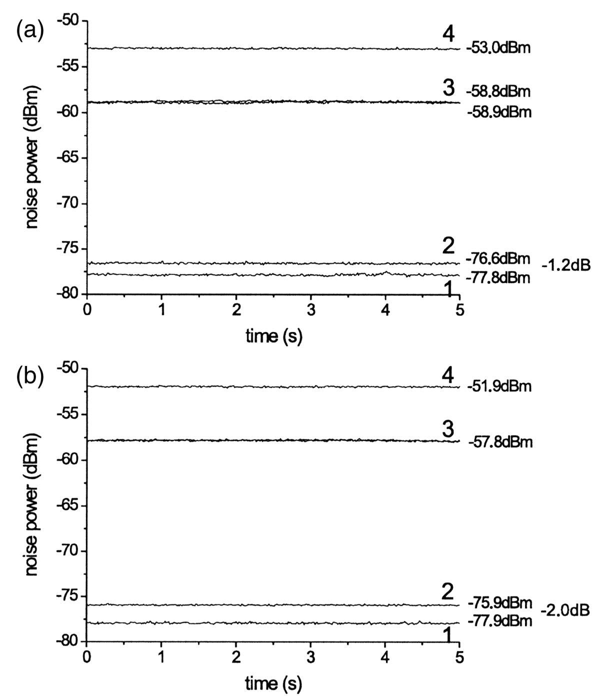

Fig. 2. Correlations of the (a) phase quadrature and (b) amplitude quadrature. In each graph the noise level (traces 1) of the correlation signal is shown together with the corresponding shot-noise level (traces 2), the noise level of the individual beams (traces 3), and the signal with the anticorrelations (traces 4). The traces were corrected by subtracting the electronic noise, which was at −84.7 dBm.

contrast and hence high efficiency of the interferometer, one has to carefully match not only the phase fronts of the light from the long and the short arms but also the temporal overlap of the pulses.

To characterize the entanglement source we first investigated the squeezing resources used to generate entanglement, using the two interferometers in the amplitude quadrature settings. In the experiment 2.1 dB and 2.4 dB of amplitude squeezing were observed for the two input beams, respectively. The level of squeezing is limited by the losses of the beams in the interferometers by use of four imperfect detectors and nonoptimum balancing.

In the next step the expected quantum correlations (anticorrelations) of the phase (amplitude) quadrature were verified. These were checked by looking at the noise level of the difference (sum) signal of each of the photocurrents of the two entangled beams for the phase (amplitude) measurement. The phase correlations are -1.2 dB below the shot-noise level [see trace 1 in Fig. 2(a)], corresponding to a squeezing variance $\langle(\delta\hat{X}^{\Omega}_{1,\pi/2} - \delta\hat{X}^{\Omega}_{2,\pi/2})^2\rangle/2 = 0.76 \pm 0.02$, whereas the amplitude correlations were at -2.0 dB [see trace 1 in Fig. 2(b)] corresponding to a squeezing variance $\langle(\delta\hat{X}^{\Omega}_{1,0} + \delta\hat{X}^{\Omega}_{2,0})^2\rangle/2 = 0.63 \pm 0.02$ (indices 1 and 2 refer to two entangled modes). The discrepancy between the noise level of the amplitude and the phase quadrature measurement comes from imperfect mode matching in the latter case. We achieved a visibility of 85%, introducing additional losses of 28% for the phase measurement. Thus, instead of -2 dB of correlations as in the amplitude measurement, we do not expect correlations stronger than -1.3 dB in the phase measurement, which agrees with the measurement results.

The noise level of the phase quadrature measurement of the individual entangled beams [traces 3 in Fig. 2(a)] is 6 dB below the corresponding sum signal of these two beams (trace 4), which is due to the quantum correlations of the entangled beams. The same applies for the noise levels of the amplitude quadrature. Note that the noise level of the individual beams is $\sim$18 dB above the shot-noise level because of the high degree of excess phase noise of the initial squeezed light introduced by self-phase modulation as well as by guided acoustic wave Brillouin scattering.[12]

In this experiment, for the first time to our knowledge nonclassical correlations in the phase quadrature have been observed directly for intense pulsed light. No additional interaction between the individual entangled beams[11] is necessary to demonstrate sub-shot-noise correlations in the phase quadrature. According to the nonseparability criterion by Duan *et al.*[13] and Simon[14] for Gaussian systems, the existence of nonclassical correlations implies that $\Delta = [\langle(\delta\hat{X}^{\Omega}_{1,\pi/2} - \delta\hat{X}^{\Omega}_{2,\pi/2})^2\rangle\langle(\delta\hat{X}^{\Omega}_{1,0} + \delta\hat{X}^{\Omega}_{2,0})^2\rangle/4]^{1/2} < 1$. Since $\Delta = (0.76 \times 0.63)^{1/2} < 1$, we can conclude qualitatively that the state is nonseparable. Because our bipartite Gaussian state is symmetric we can also quantify the entanglement of formation E_F by use of proposition 2 in Ref. 15, and we find that $E_F = 0.22 \pm 0.02$.

The implemented phase-measurement device opens the possibility of performing a variety of experiments in the field of quantum information and communication, where intense light beams are used and phase measurements are required. For example, a quantum key distribution scheme relying on quadrature entanglement[16] could be implemented without sending a local oscillator together with the signal beams. Also, the full experimental demonstration of continuous variable entanglement swapping using intense light beams seems to be possible.[2]

In conclusion, we have experimentally demonstrated a setup for measuring the quantum fluctuations of the phase quadrature of intense, pulsed light without using a separate local oscillator or a resonator. We prove sub-shot-noise resolution by resolving phase correlations below the shot-noise level for a pair of quadrature-entangled beams.

We thank N. Lütkenhaus, P. Grangier, and J. Trautner for useful discussions and A. Berger for building the electronic circuits for the phase lock. This work was supported by the Schwerpunktprogramm 1078 of the Deutsche Forschungsgemeinschaft and by the Kompetenzetzwerk Quanteninformationsverabeitung (A8). U. L. Andersen gratefully acknowledges financial support of the Alexander von Humboldt foundation.
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