Direct, Loss-Tolerant Characterization of Nonclassical Photon Statistics

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We experimentally investigate a method of directly characterizing the photon-number distribution of nonclassical light beams that is tolerant to losses and makes use of only standard binary detectors. This is achieved in a single measurement by calibrating the detector using some small amount of prior information about the source. We demonstrate the technique on a freely propagating heralded two-photon-number state created by conditional detection of a two-mode squeezed state generated by parametric down-conversion.

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The photon-number distribution is a key characteristic of every optical field. Several indicators of nonclassicality are based directly on the measurement of these statistics, including the negativity of Mandel's Q parameter and the negativity of the Glauber-Sudarshan P function [1]. Furthermore, many quantum information applications rely on light sources with well-defined photon-number distributions. For example, quantum cryptography based on single-photon protocols [2] requires the complete suppression of multiphoton components in order to guarantee security over longer distances. The ability to directly measure the photon-number distributions, then, is important both fundamentally and technologically. Two main obstacles have hindered progress in such measurements: First, photon-number-resolving detectors were devised only recently [3,4]; second, all available single-photon sensitive detectors exhibit finite efficiencies, such that intrinsic losses often mask the signatures of nonclassical states.

Currently, there are three approaches to retrieving photon-number distributions: using photon-number-resolving detectors [4]; via state reconstruction from homodyne tomography [5,6]; and using binary ("click-counting") detectors such as avalanche photodiodes (APDs) [3,7,8]. However, all of these approaches are compromised by loss and detector inefficiencies, which cause instabilities in the inversion algorithm used to reconstruct photon-number distributions from count statistics. This makes it difficult to reconstruct the quantum state of the source since the detector efficiency must be known accurately. Several photon-number-resolving detectors have high detector efficiencies, though these are usually accompanied by noise from dark counts, which also affects the fidelity of the inversion. In homodyne detection, the calibration is made yet more difficult by the need to match the modes of the quantum state with that of the local oscillator. Two distinct approaches are currently known using APDs: measurements of the mean count rate as a function of beam attenuation [7,8] and mode multiplexing to implement a photon-number-resolving detector [3]. Photon-number characteristics for classical states have been measured with APDs in the past [3,8,9], but the problem is much more intricate for nonclassical states, which do not maintain the form of their statistics when attenuated. Absolute APD quantum efficiency measurements usually require an independent measurement in which the detector response can be distinguished from external losses. Poor calibration may compromise the accuracy of methods such as the attenuation approach that rely on well-known losses. In contrast, the mode-multiplexing approach greatly increases the accuracy of detector calibration and does not require an independent measurement.

In this Letter, we propose and demonstrate a new APD-based approach to direct loss-tolerant photonic state characterization. We exploit partial *a priori* information about the photon-number distribution, typically known from the state generation process, to accurately calibrate the total loss in the channel taken by the state of interest (hereafter referred to as the signal). Thus, we generalize the idea of a self-referencing detector, originally introduced in 1977 by Klyshko [10], to different types of multiphoton states. We use a time-multiplexed detector (TMD) to emulate a photon-number-resolving detector. The measured count statistics $p(k)$ are related to the photon-number distribution $\rho(n)$ by $p(k) = \mathbf{C} \cdot \mathbf{L}(\eta) \cdot \rho(n)$, where $\mathbf{L}(\eta)$ is the matrix describing the binomial process of loss with an overall efficiency of η, and $\mathbf{C}$ is a matrix that takes into account that the TMD can resolve only up to a finite number of photons at the input [11,12]. Thus, the photon-number distribution at the source can be reconstructed from the count statistics by directly inverting these matrices or by using a maximum likelihood technique. We emphasize that we utilize this calibration technique to accomplish, from a *single* measurement set, both loss estimation and a reconstruction of the photon-number distribution at the source—allowing loss-independent state characterization for a given generation process without the need of a premeasurement of the loss. As with Klyshko's original detector calibration scheme, our approach relies on knowledge that we infer from the state generation; previous theoretical and experimental work indicates that such inferences are reasonable [6,13,14]. We emphasize that earlier theoretical work [15] has shown that for known losses such compensation procedures are always possible when the overall detection efficiencies exceed 50% (and yet smaller efficiencies can be tolerated for specific classes of states).

In general, any type of prior information can be used, but in this Letter we use the strict photon-number correlations between the modes of a two-mode squeezed state; the state of the field is

$$|\psi\rangle = \sqrt{1 - |\lambda|^2} \sum_{n=0}^{\infty} \lambda^n |n\rangle_h |n\rangle_v, \tag{1}$$

where the modes are labeled by orthogonal polarizations h and v, and λ is the parametric gain [1]. The state we wish to characterize is the horizontally polarized mode (signal arm) that is conditionally prepared by detection of the vertical mode (trigger arm). This would normally require an independent measurement of the detector parameters and channel loss but, using our technique, can be inferred directly from the state characterization data. Historically, detector calibration was accomplished by assuming that only the first two terms of the sum in Eq. (1) contribute, measuring each mode with an APD, and comparing the singles counts to the coincidence counts. We extend this idea in two ways: First, we incorporate k terms of the sum in Eq. (1), where k denotes the number of photons detected as a trigger, allowing the use of higher parametric gains and the characterization of a broader range of states; second, we detect the signal with the TMD, allowing us to see the complete count statistics of the signal, from which we then derive the losses.

The experimental setup is shown in Fig. 1. A mode-locked titanium sapphire (Ti:sapphire) laser (100 fs pulses at 800 nm and a repetition rate of 87 MHz) pumps second harmonic generation, which we subsequently filter down to a 2 nm bandwidth (FWHM). Type II parametric down-conversion (PDC) is generated in a 12 mm long z-cut KTP

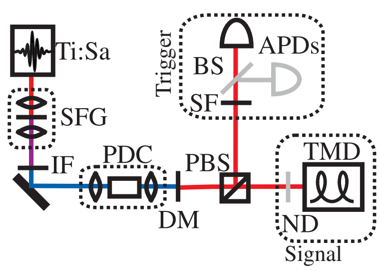

FIG. 1 (color online). The experimental setup: A mode-locked titanium sapphire laser (Ti:Sa) pumps sum frequency generation (SFG). The Ti:sapphire laser is eliminated with Schott glass filters (not shown) and the SFG bandwidth is restricted with an interference filter (IF). This is used to pump PDC in a waveguide, and the blue is removed with a dichroic mirror (DM) and Schott glass filters (not shown). The PDC photons are split at a PBS. The trigger arm (reflected) contains a spectral filter (SF) and can be detected with either a single APD or a BS and two APDs detected in coincidence. The signal arm (transmitted) is analyzed with the TMD. The additional loss (ND) was optionally placed prior to the TMD.

waveguide with 5 μW of second harmonic power. The orthogonally polarized daughter photons split into different spatial modes at a polarizing beam splitter (PBS). The trigger arm is spectrally filtered with a 15 nm filter (FWHM). Details of this high brightness, waveguided down-conversion source are presented elsewhere [16]. For single-photon generation, the trigger is a multimode fiber-coupled APD (Perkin-Elmer SPCM-AQR-13). Alternatively, for two-photon state preparation, an additional BS is inserted into the trigger arm and two APDs are detected in coincidence (3 ns coincidence window) as a trigger. A trigger is accepted only if it occurs in a well-defined 700 ps window relative to the Ti:sapphire pump pulse. The signal arm is detected with the TMD.

Our results are presented in Fig. 2. We begin with the case of single APD trigger [Figs. 2(a) and 2(c)]. Note that if an APD was used in the signal arm this setup would be equivalent to that of Klyshko. However, the use of the TMD allows us to verify the complete count statistics of the conditionally prepared state that can be used to verify the validity of our assumption that λ is small such that higher order terms of Eq. (1) are negligible. We define the "Klyshko efficiency" for our detector as [17]

$$\eta_K = p(\text{click}|\text{click}) = \frac{\sum_{i=1}^{\infty} N_i}{\sum_{i=0}^{\infty} N_i},$$

where N_i is the number events where i photons would be

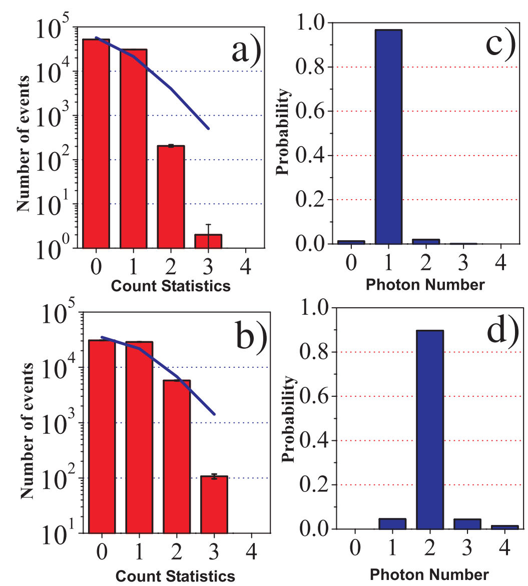

FIG. 2 (color online). Count statistics detected (logarithmic scale) using (a) single and (b) double APD as a trigger. The solid line (used to guide the eye) shows the Poisson distribution with the same mean photon number as the data (0.376 and 0.623, respectively). This line illuminates the sub-Poissonian nature of our measured statistics. The photon-number distribution obtained by using a maximum likelihood inversion with constraints $\rho(n) \geq 0$ for (c) a single- and (d) a double-APD trigger by taking into account the efficiencies, which were 37.3% and 31.5%, respectively.

registered by a TMD and $p(\text{click}|\text{click})$ is the probability of registering a click in the signal arm conditional on receiving a click in the trigger arm. Using the TMD, we are able to define the efficiency associated with single-photon triggers as

$$\eta = p(1|t=1) = \frac{N_1}{\sum_{i=0}^{\infty} N_i},$$

where $p(i|t=1)$ is the probability that i photons were registered in the signal arm given a single APD click in the trigger arm. This relation and the further relation $p(0|t=1) = 1-\eta$ can be used to deduce an overall signal efficiency of $37.3 \pm 0.1\%$ from the data. Accounting for losses, this corresponds to a single-photon conditional preparation efficiency of 97%.

In our approach, we utilize the complete conditional statistics to ascertain η from the experimentally independent measurements of the different photon numbers n. Note that, in the general case of k photons detected in the trigger arm, we expect $p(n<k|t=k)=0$ if there is no loss or detector inefficiency ($\eta=1$), due to the prior information of number correlations in the two modes. Thus, all probabilities $p(n<k|t=k)>0$ are caused solely by losses in the signal arm with

$$p(n<k|t=k) = \binom{k}{n}\eta^n(1-\eta)^{k-n}. \qquad (2)$$

In this way, we can exploit all such contributions with $n<k$ to obtain a value for the efficiency independent of all other experimental parameters such as the parametric gain λ and the loss in the trigger arm. We note that any state of light that has $\rho(n)=0$ for at least one value of n [such as a single mode squeezed state where $\rho(n)=0$ for all odd values of n] is a perfect candidate for this technique.

For the double trigger [Figs. 2(b) and 2(d)], one can calculate the losses in the signal arm in three different ways using Eq. (2). Given a two-photon detection in the trigger arm ($t=2$) and a signal efficiency η, it is straightforward to calculate the following relations between these probabilities and the efficiency in the signal arm:

$$\eta^{(0)} = 1 - \sqrt{p(0|t=2)}, \qquad (3)$$

$$\eta^{(1)} = \tfrac{1}{2}[1 - \sqrt{1-2p(1|t=2)}], \qquad (4)$$

$$\eta^{(2)} = \sqrt{p(2|t=2)}. \qquad (5)$$

To evaluate the efficiencies $\eta^{(j)}$ from our raw data, we must consider not only the losses in our case but also the limitations of the photon-number-resolving capabilities of the TMD. This is done by multiplying our click statistics by the inverse of $\mathbf{C}$ and then applying Eqs. (3)–(5). We emphasize that this matrix can be obtained from the same measurement as the characterization and no supplemental measurement is necessary. Using the above relations, we find that $\eta^{(0)} = 31.5 \pm 0.1\%$, $\eta^{(1)} = 31.0 \pm 0.2\%$, and $\eta^{(2)} = 32.1 \pm 0.2\%$. This corresponds to an average efficiency of $31.5 \pm 0.2\%$ and a two-photon conditional state preparation efficiency of 90%. The reason that these numbers differ by a degree larger than the error is because the state is not a true two-photon state but contains small contributions from higher photon numbers. Qualitatively, it is simple to see that with higher photon-number contributions $\eta^{(2)}$ would be larger than its true value because $p(2|t=2)$ would be higher than if $\rho(n>2)=0$. Simulations of our technique confirm this quantitatively. We note that this problem occurs in all calibration approaches using twin photon beams. However, in our case we are able to assess directly the validity of our assumptions by having access to the full count statistics, something that is impossible without photon-number resolution. Even with this consideration, our estimate of the signal arm efficiency (mean of the above numbers with error bars encompassing the spread of values) is the most precise direct calibration available.

To test the accuracy of our loss estimation, a neutral density filter was inserted into the signal arm before the TMD, using the double-APD-trigger configuration. The filter was calibrated to have a transmission of $13.5 \pm 0.1\%$ using the Ti:sapphire laser and a linear photodiode. Using the previously mentioned loss relations, a measurement of the efficiency was performed with and without the additional filter. The ratio of these two efficiencies gives a filter transmission of $13.8 \pm 0.1\%$. The slight discrepancy in transmittance is likely due to the spectral differences between the laser and the PDC signal photons. (Nondegenerate PDC was used, resulting in a spectrum with a different central wavelength and bandwidth from the laser. This calibration discrepancy reemphasizes the need for a more accurate way to calibrate loss than with a prior measurement using classical light.) As is expected for this case of very high loss (95.5%), the fidelity of the inverted distribution decreases to 75%. Experimental attenuations performed with the single APD trigger were able to accurately reconstruct photon-number distributions (showing >90% fidelity to a single-photon state) for a total signal loss of up to 88.0%.

An important case to study is when the prior information used (e.g. photon-number correlations) is not valid. The most extreme situation would be the substitution of uncorrelated light sources for the PDC source. We investigate this effect theoretically by mixing a coherent state into the PDC signal arm with various mean photon numbers. We find that the fidelity of our photon-number reconstruction stays above 90% for means up to 0.3 photons, independent of the loss; the additional mean number can be significantly higher than 1 photon if the efficiencies are higher than 55%. Another key observation is that decreasing prior knowledge accuracy (i.e. increasing the mean photon number of the additional coherent state) leads to inconsistent results among the efficiency measurements. We confirm this experimentally for the extreme case of completely uncorrelated light by pumping a waveguide that is not periodically poled and therefore produces no PDC but creates spurious fluorescence counts. This yields count statistics with a very low mean photon number, which we evaluate according to our previous analysis. Using the false assumption of photon-number correlations results in: (i) inconsistent loss measurements and (ii) unphysical reconstructed photon-number distributions (distributions with negative probabilities) [18].

Finally, to prove nonclassicality, we show that both our detected statistics and the inferred photon-number probabilities result in negative values of Mandel's Q parameter, which is a sign of nonclassicality. All Fock states result in $Q = -1$, and all coherent states have $Q = 0$. The count statistics detected using a single APD trigger give $Q = -0.36$, and the inferred probability distribution result in $Q = -0.97$. Moreover, using two APD triggering, the detected statistics yield $Q = -0.32$, and the inverted distribution results in $Q = -0.93$. To test the consistency of this nonclassicality, we also investigated the negativity of the P function for our data. This measure can be formulated in terms of conditions on the photon-number distributions as [13,19]

$$B(n) \equiv (n+2)\rho(n)\rho(n+2) - (n+1)[\rho(n+1)]^2 < 0,$$

where the inequality need only be satisfied for one value of n to assure the negativity of the P function. Our detected statistics do not satisfy the conditions, but our inferred photon probability distributions do. For the single [double] APD case, $B_1(0) = -0.13$ [$B_2(0) = -0.11$], once again showing the nonclassicality of the states.

In summary, we have demonstrated a method of nonclassical state characterization using mode multiplexing and standard APDs. An inversion of the counting statistics was used in conjunction with a "calibration-free" measurement of the loss in order to recreate the photon-number distribution of a heralded photon source based on the two-mode squeezed state. In the future, this technique could be extended to more general nonclassical states where distinct properties which change under the influence of loss are known. In the context of conditional state preparation, our characterization technique can also be extended where loss calibration is not obvious. By utilizing properties of known unconditioned statistics, we can estimate the losses in the system and thus obtain the loss-tolerant calibration for postselected conditioned subsets. Hence, we expect that our detection scheme will become particularly relevant for quantum information protocols such as entanglement distillation [20].

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