Multi-photon entangled states of light are key to advancing quantum communication [1, 2], computation [3], and metrology [4]. Current methods for building such states are based on stitching together photons from probabilistic sources [5, 6]. The probability of \( N \) such sources firing simultaneously decreases exponentially with \( N \) [7], imposing severe limitations on the practically achievable number of coincident photons [6]. We tackle this challenge with a quantum interference buffer (QIB), which combines three functionalities: firstly, it stores polarization qubits, enabling the use of polarization-entangled states as resource; secondly, it implements entangled-source multiplexing, greatly enhancing the resource-state generation rates; thirdly, it implements time-multiplexed, on-demand linear optical networks for interfering subsequent states. Using the QIB, we multiplex 21 Bell-state sources and demonstrate a nine-fold enhancement in the generation rate of four-photon GHZ states. The enhancement scales exponentially with the photon number; larger states benefit more strongly. Multiplexed photon entanglement and interference will find diverse applications in quantum photonics, allowing for practical realisations of multi-photon protocols.

**Introduction** — Entangling multi-particle quantum systems is an important task in quantum technologies [8], where large entangled photonic states are key to multi-party communication. They provide a distinct advantage over pairwise entanglement [9], if the cost of the multi-photon state preparation is not too high [10]. Large entangled states are also critical for obtaining a quantum advantage in cluster-state linear optical quantum computation [11] and quantum-enhanced metrology [12].

Current sources of multi-photon entangled states do not fulfill the demanding requirements of these applications, because they are primarily based on the simultaneous generation of many entangled photon pairs and their subsequent interference in linear optical networks. A prime example for this is the probabilistic generation of multi-photon GHZ states [5, 6], where \( N \) photon pairs are probabilistically combined to produce a 2\( N \)-photon state. Typically, individual sources generate entangled photon pairs with probability \( p \ll 1 \). The probability of \( N \) sources producing pairs simultaneously in an average number of \( M \) trials is then \( M p^N \); it decreases exponentially with increasing number of sources.

This scaling can be overcome by source multiplexing, which refers to the method of using multiple effective sources and feed-forward to increase generation rates. This idea has initially been put forward for the generation of single photons [13] but generalises to other states (e.g., photon pairs), given that heralding is possible. When a generation event is heralded from one source, the generated photon pair (or a part thereof) is stored in a quantum memory. This is implemented for all \( N \) sources, giving each source \( M \) chances to produce a pair; after all sources have fired, the stored states are released and processed further. In this case, the generation probability grows as \((M p)^N\), thus providing an \( M^{N-1} \) enhancement [14].

To date, source multiplexing has been demonstrated for the enhanced generation of heralded single photons [14–16]. Entangled state multiplexing, however, is still an outstanding challenge. Independently of this, the storage of polarization qubits — a requirement for the multiplexed generation of polarization-entangled states — has only been demonstrated in solid state quantum memories [17–20], which cannot yet implement entangled-source multiplexing due to low efficiencies and/or low rates. Finally, time-multiplexing, which exploits many time bins as opposed to the more typical spatial modes as qubit register, has been shown to facilitate the resource efficient implementation of linear optical networks; by cycling through a loop architecture, the networks become inherently stable and homogeneous [21–23]. But they have so far been limited to interfering consecutive time bins. Examples of such networks have been used for generating GHZ states, however without the rate enhancement provided by source multiplexing [24, 25].

Here, we introduce the quantum interference buffer (QIB), a device that brings together all the above functionalities and enables the multiplexed generation of multi-photon entangled states. The QIB combines efficient and low-noise polarization qubit storage, entangled-source multiplexing, and feed-forward time-multiplexed linear optical networks. It thus facilitates the resource-efficient generation of large entangled states while at the same time keeping the experimental overhead constant regardless of the size of the generated states.

**Operating scheme** — The operating scheme of the QIB is summarised in Fig. 1. We focus on the generation of 2\( N \)-photon shared GHZ states, where Alice keeps \( N \)
FIG. 1. Operating principle of the QIB and experimental setup. a, Spatial realisation of source-multiplexed GHZ state generation. A 1 × N switchyard routes a train of pump pulses to N entanglement sources. First, source 1 is pumped. When a Bell-pair is generated, one photon is detected, whilst the other is stored in a quantum memory. Then the pump is switched to source 2; this optimises the usage of pump power. This repeats until all N sources have fired, after which the stored photons are released and interfere pairwise in a linear optical network, whose outputs can be routed to N Bobs for entanglement sharing. The detection of a specific output pattern then confirms the successful generation of a GHZ state. b, The unique capabilities of the QIB facilitate a source-multiplexed and resource efficient setup. A stream of pulses pumps a single physical Bell-state source. The detection of a herald photon triggers the storage of its sibling in the QIB, thus implementing source multiplexing. Subsequent detection of the next photon switches the QIB to interference, after which one photon leaves the QIB whereas one is buffered; this is the time-multiplexed optical network. In this way, multi-photon GHZ states are subsequently built up. After the herald detection and interference, the final photon is released from the QIB. Post-selection of one photon in each output time bin confirms the presence of the complete 2N-photon GHZ state. c, Experimental setup comprised of entangled photon pair source (green), qubit interference buffer (blue), and polarization analysis stages (yellow). For more information see the text and supplemental.
Results for storage of single and entangled qubits for up to 1 μs. a, The post-storage fidelity of polarization qubits decreases slowly, in particular the minimum average fidelity is (97.8 ± 1.0) % in the region up to 20 roundtrips relevant for GHZ state production. The model curves include multipair emissions, roundtrip losses, and a small imperfection in the quarter-wave plate angle of 0.27°. b, The memory efficiency of the QIB falls exponentially with a roundtrip efficiency of (90.57 ± 0.06) % extracted from the fit, slightly below the single roundtrip value of 91.7 % (see supplemental) due to imperfect spatial mode matching at large storage times. The total efficiency is the efficiency to retrieve a photon at the polarization analysis stage (3) given one was present in the fibre at the source (1), cf. Fig. 1. The memory efficiency has the losses that accrue even without activating buffer function (circulator, coupling losses) removed. c, HOM visibilities resulting from the interference of two heralded photons one of which has been stored in the buffer for the given storage time before being interfered with the other. The model curve assumes no imperfections except for asymmetric losses from storing one photon that is, the stored photon expects loss in the QIB which the freshly generated one does not. All error bars originate from Poissonian counting statistics, apart from average fidelity, which comes from the standard deviation of the fidelity across the six input basis states. Here as in all data presented, no accidental counts have been subtracted.

Results – We characterise the QIB operation through three complementing measurements: firstly, the storage of polarization qubits; secondly, the storage of polarization entanglement; finally, Hong-Ou-Mandel (HOM) interference between a stored and a freshly produced photon. For polarization-qubit storage, we generate unentangled photon pairs, project one of the photons into a user-defined polarization state, store it for a certain number of roundtrips, and perform a polarization tomography after releasing it. For the storage of entanglement, we generate polarization Bell pairs, store one partner of the pair, release it, and then perform a joint polarization tomography on both photons. In the former case, we find an average fidelity of (99.7 ± 0.1) % and a process fidelity of (97.72 ± 0.03) % for a storage time of 13.16 ns. In the latter case, the fidelity is (93.8 ± 0.1) %, limited by the initial entanglement fidelity of the source, see Fig. 2a. The storage efficiency is shown in Fig. 2b, from which we extract a storage efficiency of (90.57 ± 0.06) %. Finally, we measure the HOM interference visibilities between subsequently generated photons, see Fig. 2c. After one roundtrip, we observe a visibility of (94.5 ± 1.8) % at low pump power, limited by the source [26]. Note that Fig. 2c has been measured at high pump power for convenience. The decrease in HOM visibility as a function of the number of roundtrips in the QIB is fully explained by the imbalanced losses between the stored and the freshly generated photon.

Next, we demonstrate the enhanced generation of four-photon GHZ states. Fig. 3a shows the main results of this Letter. The generation rate increases with the number of multiplexed sources, up to a maximum nine-fold increase for 21 multiplexed sources. At the same time, the state fidelity stays basically constant as a function of the number of multiplexed sources. This demonstrates that source multiplexing with the QIB has no detrimental effect on the generated states. Fig. 3b highlights the increase in four-photon GHZ state generation rate as a function of pump power. This increase is faster for a larger number of multiplexed sources, which means that for a given target generation rate, the pump power can be kept at a lower level thus reducing unwanted multi-photon components. The effect of this reduced pump-power requirement is seen in Fig. 3c, where the state fidelity for a given generation rate is always higher when multiplexing more sources.

Discussion – We have demonstrated that the QIB facilitates the multiplexed generation of multi-photon entangled states, increasing both the generation rate as well as the state fidelity. Here, we will compare the QIB performance to that of other systems. All-optical polarization-independent storage loops have been proposed before, but not yet implemented [27]. The storage efficiency of the QIB of (90.57 ± 0.06) % is currently slightly below that of...
other, polarization-selective storage loops [14]. This is due to the double-pass through the polarizing beam splitter and number of mirrors used. In contrast, polarization-qubit storage has been realised in solid-state quantum memories. The QIB compares favourably to those, both in terms of storage fidelity and operation bandwidth and wavelength (c.f. [17–20], also see the supplemental). The lifetime of the QIB is shorter than for many solid-state quantum memories but suffices for the applications considered here. We also want to highlight the outstanding noise performance of the QIB. An established benchmark for this is the $\mu_1$ number, the ratio between the number of noise photons and the memory efficiency [28]. The QIB features a $\mu_1$ of $2.2 \times 10^{-6}$, with the dominant noise source being dark counts in the detectors; a technical noise source rather than a fundamental limitation. Finally, a limited EOM switching speed inside the QIB has limited our demonstration to four-photon GHZ states; however, the operation principle of the QIB extends to larger states, and we expect more significant improvements for larger states, e.g. a factor of up to $10^5$ for 12-photon GHZ states.

**Conclusion** – We have introduced a novel scheme for the source-multiplexed and resource-efficient generation of multi-photon entangled states, the QIB. We have demonstrated polarization qubit storage with fidelities exceeding 99%, and a nine-fold increase in the generation rate of four-photon GHZ states. We predict significantly larger improvements for larger states. In addition, the QIB is not limited to GHZ-state generation, but can be adapted to realise other classes of entangled states, e.g. tensor network states. These results put the QIB at the forefront of approaches aiming to generate large, shared entangled states for quantum information applications, and we expect the QIB to have a large impact on future realisations of state generation protocols.

**Acknowledgements** – This work was supported by the ERC project QuPoPCorn (no.725366), the ERC Synergy grant BioQ, the Alexander-von-Humboldt Foundation, the EU projects AsteriQs and Hyperdiamond and the BMBF projects DiaPol and Nanospin.

**Methods**

**Source of entangled photon pairs.** The entangled pairs are produced in a potassium titanyl phosphate waveguide (ADVR Inc.) embedded in a Sagnac loop, pumped by a femtosecond Ti:Sapphire laser (Mira 900f, Coherent). Full characterization of the source is provided
in [26]. The waveguide and pump bandwidth have been engineered to produce nearly single-spectral-mode photons, with an upper bound for the single-modeness from the measured joint spectral intensity of 98%. In the current experiment the source has a Klyshko efficiency [29] of 33% to 38% on the heralding (Alice, signal) side, and 13% to 20% on the QIB (Bob, idler) side, including transmission through the QIB. Variations are due to alignment and differences in detector efficiency between the various channels. The source can produce Bell pairs with fidelity to the maximally-entangled state up to 96%, limited by imperfect waveguide end-facet coatings [26]. The detectors are custom superconducting nanowire single-photon detectors from Photon Spot Inc. They have 75% system detection efficiency with a dead time (98% recovery) of 13 ns.

Quantum interference buffer. The QIB acts as an all-optical polarization-insensitive memory and reconfigurable linear optical network. The QIB can rapidly switch between three functions: (i.) the store-release function, in which an incoming photon enters the Sagnac loop and any photon in the QIB exits it; (ii.) the buffer function, in which any photon in the QIB stays there and any incoming photon leaves the QIB unchanged; and finally (iii.) the interference function, in which the incoming photon and the stored photon interfere at a PBS-like interaction.

The setup of the QIB is presented in Figure 1 in blue and comprises a retro-reflective delay line connected to a Sagnac loop. Light that enters the Sagnac loop is split in polarization at the PBS and the two components counter-propagate around the loop. To realize the store-release function, the EOM is left off. Since there is no polarization change, any light entering the Sagnac loop from the source gets transferred to the retro-reflective line and vice versa.

For the buffer function, the EOM is switched on while the photon is in the retro-reflective line. Then, after re-entering the Sagnac loop, both polarizations are flipped ($|V\rangle \rightarrow |H\rangle$ and $|H\rangle \rightarrow |V\rangle$), such that the photon is returned to the retro-reflective line. To undo this flip, light that enters the retro-reflective line also undergoes a polarization flip thanks to a static quarter-wave plate at 45° passed twice. So long as the EOM remains off, the photon passes between the retro-reflective arm and Sagnac loop and remains in the QIB.

For interference, two photons enter the Sagnac loop, one fresh from the source and one from the retro-reflective line. Now the EOM is turned off after the clockwise-propagating components of the two photons have passed it, such that these receive a polarization flip but the counter-clockwise components do not. This performs the same mode-sorting action as a static PBS. For GHZ state generation, each photon now has a 50% chance to leave the QIB or return to the retro-reflective arm, leading to the 50% post-selection probability. To release the photon that returned to the retro-reflective arm, the EOM is left off such that the photon couples out on the next round trip. For larger GHZ states, post-selection works when for each heralded photon that enters the QIB, one photon is found to exit on the next round trip.

GHZ state measurements. We evaluate the fidelity via the population and coherence as detailed in the supplement. See example raw count data and the population and coherence in the supplement.

Generation of tensor network states including cluster states. In the main text, we have described the generation of GHZ states. The most general states generated by the setup are one-dimensional tensor network states with bond dimension two as each of the photons emitted from the setup has interfered with its preceding and succeeding photons at the beamsplitter. As detailed in the supplement and following the analysis of [30], such a sequence of nearest-neighbour gates acting on qubits leads to a tensor network with a bond dimension equaling the physical dimension, which is two in this case. In particular, the setup can be modified straightforwardly to generate linear cluster states. We also detail the generation of cluster states using our setup in the supplement.
Supplementary material: Exponential enhancement of multi-photon entanglement rate via quantum interference buffering

Here we present detailed information about experimental components and the theoretical design and analysis. The supplementary material is structured as follows. In Section S1, we describe the design of the quantum interference buffer (QIB) used to store and interfere the photons including a comparison with other quantum buffers and memories. Next in Sections S2 and S3, we present the schemes for generating GHZ and cluster states using the QIB in tandem with the source of entangled photon pairs. We show in Section S4 that the scheme leads to an exponential increase in the probabilities of the generating $N$-photon entangled states. Section S5 presents further details about the numerical simulations that the experimental data is compared against. Section S6 provides details about experimental aspects including feedforwarding, timing, Pockels cells and the loss budget. We conclude in Section S7 with additional experimental data.

S1. ACTION OF QIB

As described in the main text, the QIB includes the two actions of firstly a fast reconfigurable polarization beam splitter and secondly a quantum buffer. Here we describe the three functions that the QIB can perform. These include (i.) the store-release function, in which an incoming photon enters the buffer and any photon in the buffer exits it, (ii.) the buffer function, in which any photon in the buffer stays in the buffer and any incoming photon leaves the QIB unchanged and finally (iii.) the interference function, in which the incoming photon and the stored photon interfere at a PBS-like interaction.

In more detail, the setup of the QIB, depicted in Figure S1a, comprises two key components: a fast reconfigurable PBS (fast PBS, depicted as dashed box) and a delay line. The fast PBS is a four-port device. The two input ports are labelled ‘in’ and ‘from’ representing the physical input and the mode arriving from the delay line respectively. Likewise, the two output ports are labelled ‘out’ and ‘to’ respectively, for the physical output and the port leading towards the delay line. The fast PBS is built out of one fixed PBS, two fixed-angle half-wave plates $w_1$ and $w_2$, and two fast programmable electro-optic modulators (EOMs) $eom_1$ and $eom_2$. The PBS and the EOMs are placed in an optical loop. In the figure, the counterpropagating paths around the Sagnac loop have been spatially separated for clarity. In other words, although the figure depicts two spatially separated paths for the clockwise and anti-clockwise cycling light and the two EOMs acting on the spatially separated paths, in the implementation these two paths coincide and the action of the two EOMs is replaced by that of a single EOM with two different settings at different times. For clarity, the setup is unravelled into the equivalent circuit depicted in Figure S1b, in which the left to right motion represents the passage of time.

Both wave plates are fixed at $45^\circ$, which swaps the polarization modes but leaves the light otherwise unchanged. Different settings of the EOM toggle different operating functions of the QIB. We are interested in the following three functions of the QIB:

1. If EOM $eom_1$ is turned on to act as half-wave plate at $45^\circ$, performing a polarization swap, but $eom_2$ is turned off, then fast PBS effects a PBS transformation between the incoming light (‘in’) and the light stored in the buffer (‘from’). Specifically, the horizontal and vertical polarizations see the different transformations:

$$|H_{\text{in}}\rangle \rightarrow |H_{\text{to}}\rangle, \quad |H_{\text{from}}\rangle \rightarrow |H_{\text{out}}\rangle,$$
$$|V_{\text{in}}\rangle \rightarrow |V_{\text{out}}\rangle, \quad |V_{\text{from}}\rangle \rightarrow |V_{\text{to}}\rangle.$$ (S1)
TABLE I. Different operating functions of the QIB based on different settings of the EOM.

<table>
<thead>
<tr>
<th>Setup settings</th>
<th>Operating Function (effected transformation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( eom_1 ) on</td>
<td>Interfere (PBS)</td>
</tr>
<tr>
<td>( eom_1 ) off</td>
<td>Buffer (Identity)</td>
</tr>
<tr>
<td>( eom_2 ) on</td>
<td>Store-release (Swap)</td>
</tr>
</tbody>
</table>

This is the ‘interfere’ function of the QIB.

2. If both the EOMs are turned on to perform polarization swaps, then the fast PBS effects an identity transformation between the input and the output modes. In other words, the light from the delay line is directed back to the delay line and the light from the physical input is emitted “unchanged” from the output:

\[
\begin{align*}
|H_{in}\rangle &\rightarrow |H_{out}\rangle, \\
|H_{from}\rangle &\rightarrow |H_{to}\rangle, \\
|V_{in}\rangle &\rightarrow |V_{out}\rangle, \\
|V_{from}\rangle &\rightarrow |V_{to}\rangle.
\end{align*}
\]

This is the buffer function, wherein light that is in the delay line cycles in the QIB, i.e., through the fast PBS and back into the delay line, and effectively isolated from the input light.

3. The final function is the store-release function, wherein light from the input ports of the fast PBS is swapped into the output ports without any change in the polarization:

\[
\begin{align*}
|H_{in}\rangle &\rightarrow |H_{to}\rangle, \\
|H_{from}\rangle &\rightarrow |H_{out}\rangle, \\
|V_{in}\rangle &\rightarrow |V_{to}\rangle, \\
|V_{from}\rangle &\rightarrow |V_{out}\rangle.
\end{align*}
\]

The light stored in the QIB is released and the input light enters the QIB.

Table I summarizes these three functions of the QIB. The EOMs can be programmed to toggle between the three different functions for different incoming pulses. Different functions of the QIB can be employed to generate photonic entangled states with exponentially higher likelihood as compared to usual setups as detailed in the Sections S2 and S3.

A. Comparison to quantum memories

In this section, we aim to compare the performance of our QIB with that of state of the art atomic quantum memories for polarization qubit storage. As already eluded to in the main text, our performance benchmarks (except for storage time) compare favourably with atomic memories. Some of the more recent demonstrations of polarization qubit storage are based on either electrically induced transparency (EIT) in cold caesium [19] and rubidium [20] ensembles, or atomic frequency combs (AFC) in neodymium [17] and europium [18].

A comparison of performance benchmarks is given in Table II. We note that the \( \mu_1 \) for the atomic memories are extracted from the memory efficiencies and noise properties stated in the respective publications. Also, the stated bandwidths are the operation bandwidths, which do not necessarily strictly correspond to the maximum achievable memory bandwidths.

It is to be expected that the QIB cannot compare with atomic systems with regards to memory lifetime. We note, however, that the lifetime of 131 ns (corresponding to 10 roundtrips in the QIB) is more than sufficient for the applications considered here, namely the multiplexed generation of multi-photon entangled states. The operation fidelity is state of the art and can be increased by a more precise tuning of the quarter-wave plate angle in the retro-reflective delay line (cf. Fig. 2). Finally, the QIB outperforms other memories in terms of bandwidth, storage efficiency, and noise performance. We note that this is to be expected, since the bandwidth is not limited by an atomic level structure, and there is virtually no noise generated inside the QIB due to the absence of a strong control field. As a sidenote, we want to add that the QIB can, in principle, operate at any given wavelength (telecommunications wavelength in our case); this is in contrast to atomic memories, the operation wavelength of which is governed by the atomic level structure of the underlying material system and which, for the memories considered here, ranges between 580 nm and 880 nm.

S2. GENERATION OF GHZ STATES

GHZ states of \( 2N \)-photons can be generated using \( N \) sources of entangled photon pairs, which are stitched together using PBSs and post-selection [5, 6, 24, 31–33]. As we have shown, a similar scheme can leverage the time-multiplexing enabled by the QIB to generate GHZ states with an exponential \((\propto N)\) increase in the generation likelihood.

Our setup for generating GHZ states comprises two main components as depicted in Fig. 1. Firstly, we use a high-performance source of entangled photon pairs based on a hybrid integrated-bulk design that we have reported in Ref. [26]. The source combines the advantages offered by integrated sources, namely higher brightness and low-power operations, with those of bulk sources, namely high output-state quality and coupling efficiency. The source is used to generate light in the state \( \sqrt{p}|HH\rangle + \sqrt{1-p}|VV\rangle/\sqrt{2}, \) i.e., entangled photon pairs with probability \( p \) and vacuum otherwise.

The source is connected to the QIB, which allows for time-multiplexing as follows. One mode (say the signal mode) of each pair is measured at a polarization-selective detector and either no photon is detected or a single photon is detected, in which case the polarization \( (H \text{ or } V) \) is known. Multi-photon emissions, in the first place kept low by pumping the source weakly, and in the second place partially obviated by removing the cases where both
TABLE II. Comparison of performance benchmarks of state of the art atomic memories for polarization qubits and our QIB.

<table>
<thead>
<tr>
<th>Paper</th>
<th>Type</th>
<th>Bandwidth</th>
<th>Efficiency</th>
<th>Lifetime</th>
<th>$\mu_1$</th>
<th>Fidelity</th>
</tr>
</thead>
<tbody>
<tr>
<td>[17]</td>
<td>AFC in Nd</td>
<td>30 MHz</td>
<td>$\approx 10%$</td>
<td>not stated</td>
<td>not stated</td>
<td>0.999</td>
</tr>
<tr>
<td>[18]</td>
<td>swAFC in Eu</td>
<td>$\approx 1$ MHz</td>
<td>$4%$ at 500 ps</td>
<td>not stated</td>
<td>0.275</td>
<td>0.885</td>
</tr>
<tr>
<td>[19]</td>
<td>EIT in Cs</td>
<td>2.5 MHz</td>
<td>$69%$</td>
<td>$\approx 15$ ps</td>
<td>7.2 $\cdot 10^{-4}$</td>
<td>0.977</td>
</tr>
<tr>
<td>[20]</td>
<td>EIT in Rb</td>
<td>$\approx 0.4$ MHz</td>
<td>86 $%$</td>
<td>3 ps</td>
<td>1.7 $\cdot 10^{-3}$</td>
<td>0.996</td>
</tr>
<tr>
<td>This work</td>
<td>QIB</td>
<td>0.52 THz</td>
<td>91 $%$</td>
<td>131 ns</td>
<td>2.2 $\cdot 10^{-6}$</td>
<td>0.997</td>
</tr>
</tbody>
</table>

The abbreviation swAFC signifies the full spin-wave storage protocol for an AFC on-demand memory. The $\mu_1$ number is the ratio between noise photons and memory efficiency, an established performance benchmark for quantum memories. For more information, see the text.

$|H\rangle$ and $|V\rangle$ detectors fire in the same mode, are ignored in the subsequent analysis. The other mode, i.e., idler, of the pairs is impinged at the input port of the QIB and the time delay introduced by the delay line is set equal to the time delay between subsequent pulses emitted from the source.

The first time a photon is detected in the signal mode, this information is fed forward to the EOMs in the QIB, which toggle it to the store-release function, so the photon from the entangled pair is directed towards the delay line. Until another photon is detected in the signal mode, the QIB is set to the buffer function. This way, the photon sent into the delay line stays cycling in the QIB.

The next time a photon is detected in the signal mode, the QIB is switched to the interference function, and the stored light interferes with the idler mode of the generated photon pair. The interference results in photon pairs with same polarization having their photons emitted into different outputs (one into ‘out’ and other into ‘to’) of the QIB but pairs with orthogonal polarization are emitted into the same output port. That is, either both bunch into ‘out’ or both into ‘to’. Only those events are post-selected in which one photon is detected in each mode. Consequently, only those two terms in which all photons have identical polarization survive and the resulting state is a GHZ state.

The exponentially higher probability in $N$ results from the fact that photons generated in many different pulses can be made to interfere together. If the setup losses allow for the cycling photon to be stored for $M$ roundtrips, then there is an $\approx M^{N-1}$-fold increase in the $2N$-photon generation likelihood. One way to see this is that each of the $N-1$ interference events is $M$ times more likely to occur. A detailed analysis of the generation probabilities is presented in Section S4.

**S3. GENERATION OF LINEAR CLUSTER STATES**

Here we show that the QIB can be additionally exploited to generate linear cluster states [34] with exponentially higher likelihood (in the number of photons) as compared to the situation without multiplexing. We use a scheme similar to that of Pilnyak et al. [25]. More specifically, the PBS in the scheme of [25] is replaced by our QIB, which is triggered by signal photons, and the optical loop is replaced by our delay line.

As we describe in the remainder of this section, a straightforward implementation of the scheme requires the setup to have an extra EOM to act as a half-wave plate at 22.5$^\circ$ when there is a photon incoming and to switch off when the QIB is to act as a buffer (See Section S3 A). The extra EOM is required to be placed after the delay line and in the ‘from’ port of the QIB as depicted in Figure S2. This EOM can be replaced by a static wave plate at the cost of reduced cluster-state generation rates (See Section S3 B).

To generate linear cluster states, the source generates photon pairs in the state $|00\rangle + \sqrt{p} |HH\rangle$ in the left and right modes. Here and henceforth we omit normalization for ease of notation. The left-going modes are detected and the detection events are only used to herald the horizontally-polarized photon in the right mode. The right-going modes impinge at the QIB after passing through a half-wave plate ($w_1$) at 22.5$^\circ$ so only $|P\rangle = |H\rangle + |V\rangle$ photons arrive at the QIB.

The first detection of a photon in the left mode toggles
the QIB to the interference function, and the reflected (‘out’) mode is discarded. This way, there is an $|H\rangle$ photon in the loop with probability 1/2. The QIB is then set to the buffer function so the $|H\rangle$ photon cycles in the delay line and QIB. As soon as the next photon is detected in the left mode, the QIB is toggled to the interference function and the EOM $p_3$ outside the loop is switched on to its half-wave voltage at 22.5°. After this interference, the QIB is switched back to the buffer mode and $p_3$ is turned off till the next detection event at the left mode. Post-selecting single-photon detection events gives the required linear cluster state.

A. Cluster state generation with additional EOM

Here we present more details about the cluster state generation procedure. The effective optical circuit for constructing the cluster states is depicted in Fig. S3. We will only consider the modes in which the source emitted a photon pair as the remaining events are post-selected out via heralding. The first interaction is between the horizontally polarized photon $|H_1\rangle$ from the buffer and the incoming $|H_2\rangle$ photon after these have undergone a polarization rotation to their respective plus states $|P_1\rangle = |H_1\rangle + |V_1\rangle$. Thus, the incoming state at the PBS is the $|P_1P_2\rangle$ state, or

$$|H_1H_2\rangle + |H_1V_2\rangle + |V_1H_2\rangle + |V_1V_2\rangle.$$ (S4)

Under the action of the PBS, this state changes to

$$|H_2H_1\rangle + |H_2V_2\rangle + |V_1H_1\rangle + |V_1V_2\rangle = |H_1H_2\rangle + |H_2V_2\rangle + |H_1V_1\rangle + |V_1V_2\rangle.$$ (S5)

As we choose states in which only one photon is emitted in each mode, after this post selection, we need only consider

$$|H_1H_2\rangle + |V_1V_2\rangle.$$ (S7)

The first qubit is emitted from the system and the second is stored in the buffer till the next $|H_3\rangle$ qubit arrives. Just before the arrival of the third qubit, the polarization rotation at the output of the second mode converts the state into a two qubit cluster state

$$|H_1H_2\rangle + |H_1V_2\rangle + |V_1H_2\rangle - |V_1V_2\rangle.$$ (S8)

Before interacting, the third qubit sees a polarization rotation, which gives

$$(|H_1H_2\rangle + |H_1V_2\rangle + |V_1H_2\rangle - |V_1V_2\rangle)(|H_3\rangle + |V_3\rangle),$$ (S9)

which after the action of the PBS and post selection turns to

$$|H_1H_2H_3\rangle + |H_1V_2V_3\rangle + |V_1H_2H_3\rangle - |V_1V_2V_3\rangle.$$ (S10)

Once the third mode is acted upon by a polarization rotation, this state is

$$|H_1H_2H_3\rangle + |H_1H_2V_3\rangle + |H_1V_2H_3\rangle - |H_1V_2V_3\rangle + |V_1H_2H_3\rangle + |V_1H_2V_3\rangle - |V_1V_2H_3\rangle + |V_1V_2V_3\rangle.$$ (S11)

which is exactly the three-qubit cluster state.

One can show more rigorously that the generated state is a linear cluster state because the input qubits are all in the $|P_i\rangle$ state. The circuit building block is a PBS followed by a half-wave plate at 22.5°, and this building block has the action of a controlled-phase gate on the $|P_iP_{i+1}\rangle$ qubits after post selection as can be seen from the transformation from Eq. (S4) to Eq. (S8). This controlled-phase building block acts repeatedly on neighboring sites thereby giving a linear cluster state [34].

From arguments similar to those presented in the main text and Sec. S4 A, time-multiplexing is expected to give a $M^{N-1}$-fold enhancement in the success probability of generating an $N$-mode cluster state if the circuit losses allow for $M$ round-trips without substantial loss in fidelity.

B. Replacement of EOM with a static wave plate

The EOM that implements 45° rotations in the cluster state procedure can be replaced with a static wave
Using a static wave plate instead of an EOM, the enhancement in the success probability because of time-multiplexing is expected to drop from $M^{N-1}$ to $(M/2)^{N-1}$. This is because only half of the $M$ feasible round-trips, only half are relevant and the other half are ignored. In the static case there are some temporal modes (when the counter is odd valued) in which a photon is emitted from the setup but this photon is not a part of the linear cluster state. Therefore, events involving these temporal modes would need to be discarded.

In either of these two operations, an important difference between our time-multiplexed GHZ generation procedure and the time-multiplexed cluster-state procedure is that the signal photons in the GHZ state are a part of the full state, but the signal photons are only used for heralding in the linear cluster state. This means that $N$ photon-pair emission events will allow a generation of $2N$-photon GHZ state but only an $N$-photon cluster state.

C. CPHASE gate with QIB

Beyond effective CPHASE gates for cluster states, we show the QIB can perform post-selected CPHASE gates in general, with the addition of a fast polarization-insensitive attenuator. We follow the prescriptions of Refs. [35, 36], giving a success probability $1/9$ for each CPHASE gate.

We describe the CPHASE operation on two general qubits, coming from $f_{\text{rom}}$ and $i_{\text{m}}$, which we label 1 and 2 respectively. The gate should apply a minus sign to the $H$ term and nothing else. The incoming state is

$$ (\alpha_1 |H_1\rangle + \beta_1 |V_1\rangle)(\alpha_2 |H_2\rangle + \beta_2 |V_2\rangle), $$

on which the mode 2 has its polarizations swapped to give

$$ (\alpha_1 |H_1\rangle + \beta_1 |V_1\rangle)(\alpha_2 |V_2\rangle + \beta_2 |H_2\rangle). $$

The PBS preserves the mode number for $H$ photons, and swaps it for $V$ photons, so the state is now

$$ (\alpha_1 |H_1\rangle + \beta_1 |V_1\rangle)(\alpha_2 |V_1\rangle + \beta_2 |H_2\rangle). $$

Now we apply a half-wave plate at $72.5^\circ$ (counterclockwise from $H$) in mode 2 to produce

$$ \left(\alpha_1 |H_1\rangle + \beta_1 \left[ -\sqrt{\frac{2}{3}} |H_2\rangle + \sqrt{\frac{1}{3}} |V_2\rangle \right]\right) $$

and

$$ \left(\alpha_2 |V_1\rangle + \beta_2 \left[ \sqrt{\frac{1}{3}} |H_2\rangle + \sqrt{\frac{2}{3}} |V_2\rangle \right]\right). $$

Then we have to attenuate with transmission $\sqrt{\frac{1}{3}}$ in mode 1 to even the terms out, giving

$$ \frac{1}{3}\left(\alpha_1 |H_1\rangle + \beta_1 \left[ -\sqrt{2} |H_2\rangle + |V_2\rangle \right]\right) $$

and

$$ \left(\alpha_2 |V_1\rangle + \beta_2 \left[ |H_2\rangle + \sqrt{2} |V_2\rangle \right]\right). $$

FIG. S4. Static circuit for generating cluster states. Notice that the input of the fourth mode is a photon $|H\rangle$ but this input is ignored as the buffer operation is incomplete. The brown circles represent the constituents of the multi-photon entangled state and the empty modes are discarded.
This requires that the HWP at 72.5° applies to mode 2, but is back to identity by the time mode 1 reaches it, and that the attenuator in mode 1 is back to full transmission by the time mode 2 reaches it. Back at the PBS, the modes are swapped in V again, and mode 2 has its polarizations flipped, to produce
\[
\frac{1}{3} \left( \alpha_1 |H_1\rangle + \beta_1 \left[ -\sqrt{2} |V_2\rangle + |V_1\rangle \right] \right)
\]
(S19)
\[
\left( \alpha_2 |H_2\rangle + \beta_2 \left[ |V_2\rangle + \sqrt{2} |V_1\rangle \right] \right). 
\]
(S20)
This is expanded to
\[
\left( \alpha_1 \alpha_2 |H_1 H_2\rangle + \alpha_1 \beta_2 |H_1 V_2\rangle + \alpha_2 \beta_1 |H_2 V_1\rangle + \beta_1 \beta_2 \left[ |V_1 V_2\rangle - 2 |V_1 V_2\rangle + \sqrt{2}(|V_1 V_1\rangle - |V_2 V_2\rangle) \right] 
+ \sqrt{2} [\alpha_1 \beta_2 |H_1 V_1\rangle - \alpha_2 \beta_1 |H_2 V_1\rangle] \right). 
\]
(S21)
Post selecting on one photon in mode 1 and one in mode 2 gives
\[
\frac{1}{3} \left( \alpha_1 \alpha_2 |H_1 H_2\rangle + \alpha_1 \beta_2 |H_1 V_2\rangle 
+ \alpha_2 \beta_1 |H_2 V_2\rangle - \beta_1 \beta_2 |V_1 V_2\rangle \right) , 
\]
(S22)
which is the CPHASE gate.

**S4. EXPONENTIAL INCREASE IN PROBABILITIES**

**A. Scaling improvement, probability**

Now we calculate the improvement in success probability of generating a 2N-photon entangled state. To focus on the production rate, let us neglect higher-order photon contributions and all losses (except storage loop losses, below). If one photon pair is produced with probability \( p \), the probability for \( N \) sources to fire simultaneously, or \( N \) subsequent pairs to be produced in the time-multiplexed case without feed-forward is \( p^N \). With the QIB, the scaling is improved as follows. Given a pair is produced in the zeroth time bin with probability \( p \), then the probability that a pair is produced in the time bin \( j + 1 \) with none before is \( (1 - p)^j p \). The first pair is different from the others because we must always wait until it is detected, rather than restart the protocol after some waiting time \( M \). Thus to find the probability of getting two pairs we sum over \( j \) as
\[
P_2 = p \sum_{j=0}^M (1 - p)^j p = p (1 - (1 - p)^{M + 1}) \approx p^2 (M + 1).
\]
(S23)
Now to produce \( N \) pairs, allowing \( n \) chances for each pair the probability is
\[
P_N = p \left( \sum_{j=0}^M p (1 - p)^j \right)^{N - 1} \approx p [p (M + 1)]^{N - 1} \] (S24) for small \( p \). Thus for \( n + 1 = 1/p \), the exponentially small probability of producing \( N \) pairs in enhanced by an \( (M + 1)^{N - 1} \) factor.

With losses in the delay line, we no longer have the same factor of enhancement but we have a much more favourable base \( (1 - p) \) instead of \( p \) in \( p^N \). Each photon sees the roundtrip loop loss \( j + 2 \) times, once for coupling in, once for interference, and \( j \) times while waiting for the next pair. Here we assumed that the waiting is reset as soon as the next pair is detected; we do not wait \( M \) steps every time. The last photon only sees two roundtrip losses since it is coupled out directly after interference. Given a roundtrip loss \( \eta \), the probability of entangling \( N \) photon pairs is
\[
P_N \approx p^N \eta^2 \left( \frac{1 - (1 - p)^{M + 1} \eta^{M + 1}}{1 - (1 - p) \eta} \right)^{N - 1} 
= p^N \eta^2 \left( \frac{1 - (1 - p) \eta^{M + 1} + p (M + 1) \eta^{M + 1}}{1 - (1 - p) \eta} \right)^{N - 1} .
\]
(S25)
In the large \( M \) limit we find
\[
P_N = p^N \eta^2 \left( \frac{1}{1 - (1 - p) \eta} \right)^{N - 1} . 
\]
(S26)
For a broad range of losses below a certain threshold, this scaling is better than the scaling without a QIB. We simply set
\[
p^N < p^N \eta^2 \left( \frac{1}{1 - (1 - p) \eta} \right)^{N - 1} \quad (S27)
\]
to find, for large enough \( N \) that \( \frac{N - 1}{N} \approx 1 \) and in the small \( p \) limit,
\[
\eta > \frac{p - 1 + \sqrt{p^2 - 2p + 5}}{2} \approx 0.62, \quad (S28)
\]
which is easily met in our setup. Comparing to a time-multiplexing setup without buffering or feed-forward, which suffers the same \( \eta^2 \) losses per photon, the advantage is for
\[
p^N \eta^2 < p^N \eta^2 \left( \frac{1}{1 - (1 - p) \eta} \right)^{N - 1} , \quad (S29)
\]
\[\rightarrow \eta > 0.\]
The QIB thus enhances the probabilities of producing a 2N-photon entangled state.

**B. Rate improvement**

The above comparison of probabilities shows a distinct improvement in generation probability using source multiplexing with the QIB, but unlike the spatial case this
procedure takes more than a single laser pulse. Thus we now examine the time taken by the QIB to generate $N$ photon pairs, and show it still provides an overall rate advantage over spatial- and temporal-schemes without multiplexing.

With the source clock frequency $f$, in the spatial case the waiting time is just one pulse, so the rate per second is $R = fp^N$. In the time-multiplexed case without feedback the average waiting time in number of pulses is

$$t_{TM} = \frac{\sum_{q=1}^{N} p^{q-1} q (1 - p) + p^N N}{\sum_{q=1}^{N} p^{q-1} (1 - p) + p^N}.$$  \hfill (S30)

Then the rate per second is $R_{TM} = fp^N / t_{TM}$. With small $p$, most attempts end immediately in failure, making $t_{TM} \approx 1$, and thus making the rate very close to the spatially-multiplexed version.

In the QIB case the average waiting time is (see below for a definition of $P_1$)

$$t_{QIB} = \frac{1}{p} + \frac{\sum_{q=1}^{N-1} p^{q-1} ((q - 1)(M) + M) (1 - P_1) + P_1^{N-1} (N - 1)\langle M \rangle}{\sum_{q=1}^{N-1} p^{q-1} (1 - P_1) + P_1^{N-1}}.$$  \hfill (S31)

The first term is the average waiting time for the first pair, which is independent of any losses or the maximum roundtrip number $M$. For the subsequent pairs, the average wait time per pair $\langle M \rangle$ is related to the maximum wait time per pair $M$ by

$$\langle M \rangle = \frac{\sum_{j=0}^{M-1} (j + 1)(1 - p)^j p}{\sum_{j=0}^{M-1} (1 - p)^j p}.$$  \hfill (S32)

This means, for the cases where we get a photon pair at all, we get it on average after $\langle M \rangle$ steps. Where we get no photon, we wait $M$ steps then restart the protocol.

This $t_{QIB}$ is the longest waiting time, (still $\approx 1$ for reasonable parameters), but the probability $P_N$ of producing $N$ pairs is much improved as seen above. The probability of getting a single pair in $M$ time bins is

$$P_1 = \sum_{j=0}^{M-1} (1 - p)^j p = 1 - (1 - p)^M.$$  \hfill (S33)

In contrast to the previous section, here we account for the first photon in the waiting time, such that $P_N = P_1^{N-1}$ rather than $P_N = pP_1^{N-1}$. The rate with the QIB is then

$$R_{QIB} = fP_N / t_{QIB}.$$  \hfill (S34)

Adding losses in the delay line as in the above section, we find the rate for the standard time-multiplexed case is $R = f (p^N)^N / t_{TM}$. For the QIB, the probability of
generating $N$ pairs is

$$P_N = p^{N-1} \eta^{2N} \left( \frac{1 - (1 - p)^N}{1 - (1 - p)\eta} \right)^{N-1}, \quad (S35)$$

which is again modified from Eq. (S25) to put the first photon pair into the waiting time.

The $P_1$ and waiting time are however unmodified, they depend only on the (assumed lossless) heralding signal, not the (un-measurable) loss of photons within the loop. Then the generation rate with losses included is still given by Eq. (S34), but with $P_N$ updated to the lossy version.

For which losses does time-multiplexing and feed-forward provide an advantage over spatial multiplexing? In general this depends on the number of desired pairs $N$, the pair production probability $p$, and the chosen maximum waiting time $M$. In fact the waiting time can be optimised for each parameter set, and we show in Fig. S5 the advantage of using multiplexing with optimised $M$ over non-multiplexed cases. We show the multiplicative increase in rate for various pumping strengths and loop efficiencies. Of course multiplexing is more effective for less lossy loops, but the advantage depends only weakly on pump power.

C. Expected absolute rates

We have demonstrated almost an order of magnitude increase in the generation rates for four-photon GHZ states. The true power of the QIB, however, becomes apparent when going to larger states; something we couldn’t do in the current experiment due to technical limitations.

In Fig. S6, we calculate the expected detection rates for large GHZ states for both the standard approach of parallel sources, and the QIB. The current state of the art is the generation of 12-photon GHZ states [6]. Using the efficiency values from this publication, we find a six orders of magnitude increase in the detection rate of 12-photon GHZ states when using the QIB, totalling about 100 detected states per second.

D. Entanglement visibility

Instead of improving the rate for a constant $p$, it is also possible to improve the entanglement visibility by reducing $p$, while keeping the $2N$-photon rate constant. We neglect the waiting time here to keep the formulas readable, but it would be only a small correction lowering the visibility of the feed-forward source. For equal rates of multi-photon production (in the large $M$ limit for the QIB), we set

$$p_S^N = p_{TM}^N \eta^{2N} = p_{QIB}^N \eta^{2N} \left( \frac{1}{1 - (1 - p)\eta} \right)^{N-1},$$

(S36)

for pair production probabilities for spatial multiplexing $p_S$, time multiplexing $p_{TM}$ and for the QIB $p_{QIB}$.

Thus the time-multiplexed single pair generation probability must be scaled up to $p_{TM} = p_S / \eta^2$, decreasing the entanglement visibility due to multi-pair emissions. However, the feed-forward fusion pair generation probability decreases to

$$p_{QIB} = \frac{p_S}{\eta^2} \left( 1 - (1 - p)\eta \right)^{N-1}, \quad (S37)$$

(S37)

for large $N$, which is smaller than $p_S$ again for $\eta > 0.62$. As an example a visibility increase from $\sim 25\%$ to $\sim 75\%$ is possible with the QIB and $2N = 10$ entangled photons for the same 10-fold production rate.

S5. DETAILS OF SIMULATIONS

In Fig. 2, simulations are performed with QuTiP [37, 38]. For the average fidelity we assume single qubits enter the loop and are subject to loss, a small rotation due to an imperfect quarter-wave plate (off by $0.27^\circ$). Then the process fidelity is directly found from the average fidelity as $F_{\text{proc}} = 3F_{\text{avg}} - 1$ [39–42]. For the entangled pairs simulation we additionally take into account the full statistics of the down-conversion source, the roundtrip loss of the QIB, and also the imperfect extinction ratio of the Pockels cell, allowing uncorrelated photons to be coupled into the QIB during storage of the desired photon. This causes the entanglement fidelity to decrease faster than the unentangled qubit case. Finally, for the HOM visibility we include accidental coincidences and the asymmetric
losses due to one photon of the two being stored in the QIB.

S6. EXPERIMENTAL DETAILS: ELECTRONICS, TIMING, EOM, LOSS BUDGET

Electronics — The herald (Alice) detection events are sent to a comparator (PRL-350TTL, Pulse Research Lab), which produces a rising and falling edge on separate outputs for each detection. The falling edges are sent directly to the timetagger (Time Tagger 20, Swabian Instruments). The rising edges are sent to an FPGA (Spartan6 Evaluation Board, Xilinx) which decides the EOM switching sequence based on the heralding events. For example for the four-photon GHZ state production with 11 multiplexed sources, if the FPGA gets two herald clicks 6 roundtrips apart, it will cause the EOM to turn on after the first photon enters the loop, remain on for 6 roundtrips, then turn off at the appropriate moment to enact a PBS operation between the two photons. The FPGA gets its clock from a frequency-doubled (by a Si5344, Silicon Labs) version of the photodiode output of the pump Ti:Sapphire laser which is repetition-rate stabilized to a master clock. The signals to turn on and off the EOM are again sent to a comparator, one output of which are connected to the timetagger to provide gating of the photon detection timetags. The other outputs from the comparator drive the EOM. Finally the photon detections after the QIB are also sent to the timetagger, and fourfold coincidences are found between the two herald and two QIB photons, gated by the FPGA switching signals.

Electro-optic modulator — The high-voltage driver for the EOM is from Bergmann Meßgeräte Entwicklung KG, and the RTP Pockels cell is from Leysop Ltd. The driver has a rise and fall time around 5 ns, sufficiently less than the half-roundtrip time of the QIB of 6.6 ns to allow switching between all three modes. The maximum duty cycle of the cell is around 5 %, and the maximum switching rate is 100 kHz, which limits our maximum count rates here. Drivers with faster rates of around 2 MHz are available, and it should be possible to reach 5 ns rise times with these too, increasing our maximum rate 20 fold. Of course as we scale to larger \( N \), the probability of \( N \)-fold coincidences decreases, meaning the extinction rate ratio of the EOM is around 100 : 1.

Timing considerations — The experiment runs off a common clock from the pump Ti:Sapphire laser, meaning optical and electronic timings are stable relative to one another. To allow sufficient time for the heralding signals to reach the FPGA, be processed, and set the EOM, the QIB photons are stored for 1.7 \( \mu \)s in single-mode fiber. This was chosen to allow very long storage times; for the GHZ states with 11 multiplexed sources, only 600 ns is required. Having both herald detections available to the FPGA before any photons arrive at the QIB is key to reducing the switching load: the QIB stores the first photon only when it is already known that the second has been heralded.

We set the maximum storage time \( M \) for each effective source. If the second photon is detected before the maximum storage time, they are interfered directly, and the source is made ready for the next heralding event, as in the idea of relative multiplexing [43]. The average first photon will thus be stored under half of the maximum time, and the maximum average repetition rate of the source will be the inverse of half the maximum storage time. For now the repetition period is limited by the maximum switching speed of the EOM.

Loss budget — The roundtrip loss of the QIB is accounted for with the following efficiencies: PBS (98.7 \%)²; EOM 98 \%; 9 dielectric mirrors (Laseroptik GmbH) (99.6 \%)³; self-coated spherical end mirror 99.3 \%. These give a total roundtrip transmission of 91.7 \%, which agrees with that extracted from the fits in Figs. 2 and 3. The roundtrip efficiency of (90.57 ± 0.06) \% from Fig. 2 is slightly lower than expected since for long storage times the mode is no longer perfectly refocused by our curved end mirrors. For only up to 5 roundtrips, where the largest contribution to the four-photon GHZ state rate comes from, we extract a roundtrip efficiency from Fig. 3 of (93.7 ± 0.7 \%).

S7. ADDITIONAL EXPERIMENTAL DATA

We present raw HOM dip count data in Fig. S7.

We evaluate the fidelity of our GHZ states following [44], and show the fidelity, population, and coherence in Fig. S8a. The fidelity is the average of the population and coherence, \( F = (P + C)/2 \). The population is evaluated from Fig. S8b as the counts in the first and last bins divided by the total counts. The coherence is evaluated from four such datasets (example in Fig. S8c) in the measurement basis \( |H\rangle = \cos(\theta_k)|V\rangle + \sin(\theta_k)|H\rangle \) for \( k = [0, 1, 2, 3] \). Then \( C = \frac{1}{2} \sum_{k=0}^{3} (-1)^k \langle M_k \rangle \) for the expectation value of \( M_k = \cos(k\pi/4)\sigma_x + \sin(k\pi/4)\sigma_y \).

FIG. S7. Further data for Hong-Ou-Mandel interference. Each plot lists the full-width half-maximum of the HOM dip, the storage time of the first photon \( t \) and the dip visibility, \( V = (\max - \min)/\max \).

- **a**, Best measured visibility at low pump power, showing the indistinguishability of the photon pair source.
- **b-d**, Example HOM dips at higher power for faster data collection, showing degradation as storage time is increased from 13 ns to 671 ns. The FWHM of the dip also decreases proportionally to the storage time, as the mirror that is scanned to measure the dip is the end mirror of the QIB, meaning the translation is effectively multiplied by the number of storage roundtrips.


FIG. S8. Further data for four-photon GHZ states. a, The fidelity as in Fig. 3, made up of the population and coherence terms. The coherence is lower than the population due to imperfect HOM interference and a small phase in the loop. b, Raw fourfold count data for the first data point of the population (two sources). c, Raw fourfold count data for the first data point of the coherence, giving the last term of the coherence $<M_k>$ in the measurement basis $|H\rangle + e^{i\theta_3/4}|V\rangle$. d, All four terms of $<M_k>$, making up the coherence term $C_{[44]}$.


[29] D N Klyshko, “Use of two-photon light for absolute cal-