Rapid Communications

Scalability of parametric down-conversion for generating higher-order Fock states

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Spontaneous parametric down-conversion (SPDC) is the most widely used method to generate higher-order Fock states ($n \geqslant 2$). Yet, a consistent performance analysis from fundamental principles is missing. Here, we address this problem by introducing a framework for state fidelity and generation probability under the consideration of losses and multimode emission. With this analysis we show the fundamental limitations of this process as well as a trade-off between state fidelity and generation rate intrinsic to the probabilistic nature of the process. This identifies the parameter space for which SPDC is useful when generating higher-order Fock states for quantum applications. We experimentally investigate the multiphoton regime of SPDC and demonstrate heralded Fock states up to $|n\rangle = 4$.

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I. INTRODUCTION

As a representation of a discrete and well-defined number of excitations of the quantized electromagnetic fields, photon number states or Fock states are of significant fundamental and practical interest in quantum optics. They are the building blocks from which a range of exotic states may be constructed [1], and find direct utility in metrology [2–4] and quantum information processing protocols [5]. Common to all these applications is that they become more advantageous with the size n of the Fock state $|n\rangle$. However, generating higher-order Fock states becomes a challenging task and different approaches have been investigated to generate them [6–10]. To date, the most common process is strongly pumped spontaneous parametric down-conversion (SPDC) in a nonlinear material [11–16].

This consists of a nondeterministic decay of pump photons into precisely correlated numbers of photons in two modes (signal and idler). Increasing the number of pump photons increases the chance of multiple decays, resulting in signal and idler modes with higher occupation numbers. In collinear type-II or nondegenerate type-I SPDC, these two modes are distinguishable (in polarization or frequency, respectively). The photon number correlations between the modes can be exploited to "herald" the generation of a particular Fock state: A projective measurement onto a specific photon number n of the idler mode will result in the preparation of an $|n\rangle$ photon Fock state in the signal mode (see Fig. 1).

Although heralded SPDC is the most widely used method to generate higher-order Fock states, a detailed study of the optimal parameter range beyond the special cases for heralding more than two photons or Schmidt modes [17–19] is missing. In particular, the interplay of heralding photon numbers arising from different underlying spectral modes (Schmidt modes) renders this problem highly nontrivial.

In this Rapid Communication we investigate heralded higher-order Fock state generation based on parametric down-conversion in terms of fundamental limits as well as unavoidable experimental imperfections arising from losses and spectral multimodeness. Furthermore, we compare our findings with experimental results from a periodically poled potassium titanyl phosphate (PPKTP) waveguide source which has shown high brightness [16] and which can be engineered to emit light into a single mode [20]. This implementation investigates different pump intensities to maximize the generation probability of higher-order Fock states. To simplify our analysis, we neglect higher-order nonlinear effects and time ordering [21,22].

II. THEORETICAL DESCRIPTION

We model the state generated by a type-II SPDC process with a general two-mode squeezed multispectral-mode PDC state given by

$$|\psi\rangle = \bigotimes_k \sqrt{1-|\Lambda_k|^2}\sum_n \Lambda_k^n\,|n,n\rangle_k\,, \qquad (1)$$

where k labels the modes, $\Lambda_k = \tanh(r_k)$ specifies the squeezing in dependence of the squeezing parameter r, and n is the photon number. In the scope of this work we will consider different spectral modes k. The signal or idler photon number probability p_n for each spectral mode in Eq. (1) is given by a geometric distribution. Without additional experimental effort it is not possible to distinguish the different spectral modes with standard photon detectors. Mathematically this is described by the convolution of the different spectral modes. Calculating the resulting distribution is a known problem and has been solved previously with generating functions [23]. Here, we introduce an alternative approach to derive the probability distribution exploiting the mathematical concept

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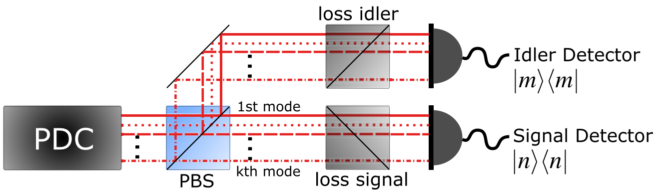

FIG. 1. Generating higher-order Fock states with type-II parametric down-conversion (PDC). The signal and idler mode are split one a polarizing beam splitter (PBS). Losses as well as spectral multimodeness are considered. For further information, see text.

of discrete phase-type distributions [24]. We are interested in the probability p_n of finding n photons in up to $K_{\max}$ modes with minimal computational effort. For our method it turns out that it is sufficient to know the vacuum probabilities $q = (q_1, q_2, \ldots, q_{K_{\max}})$ with $q_k = 1 - |\Lambda_k|^2$ for the different modes k. The probability p_n is now given by

$$p_n(q) = \alpha M^{n+K_{\max}-1} M_0, \tag{2}$$

where the matrix M is defined by

$$M = \begin{bmatrix} 1-q_1 & q_1 & 0 & \cdots & 0 \\ 0 & 1-q_2 & q_2 & \cdots & 0 \\ 0 & 0 & 1-q_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1-q_{K_{\max}} \end{bmatrix}, \tag{3}$$

with $\alpha = (1, 0, \ldots, 0)$ and $M_0 = (0, 0, \ldots, q_{K_{\max}})^T$. An even simpler solution is possible if all spectral modes have the same vacuum probability (see Supplemental Material [25]).

From now on, we will assume Gaussian functions for the pump spectrum and for the phase matching, which is a valid assumption if, for example, spectral filtering or apodized poling [26,27] is used. In this case, the squeezing parameter r_k for the kth mode is exponentially decreasing [28],

$$r_k = B\lambda_k, \quad \lambda_k = \sqrt{(1-\mu^2)}\mu^{k-1}, \tag{4}$$

where B is the optical gain that defines the squeezing strength and $\mu \in [0, 1)$ determines the effective number of spectral modes known as the Schmidt number $K = 1/\sum_k(\lambda_k^4)$. We want to stress here that this definition of the Schmidt number does not depend on the optical gain.

As the general two-mode squeezed vacuum state has perfect photon number correlations between the signal and the idler, heralding is an established method to prepare higher-order photon number states. However, the quality of this technique can be severely affected by three main challenges: fundamental limits resulting from the stochastic nature of the parametric down-conversion process, losses in the signal and idler arm, and multimodeness in the SPDC. As a first step, we start with a spectral single-mode SPDC, as a best case scenario [20], and analyze the ultimate fundamental limit resulting from the photon statistics intrinsic to the pair production process. Considering only the heralding mode (heralded mode is traced out), the geometric distribution of this thermal state reveals the maximal heralding probability for an n-photon Fock state generated by a single-mode source (cf. top inset of Fig. 2) of

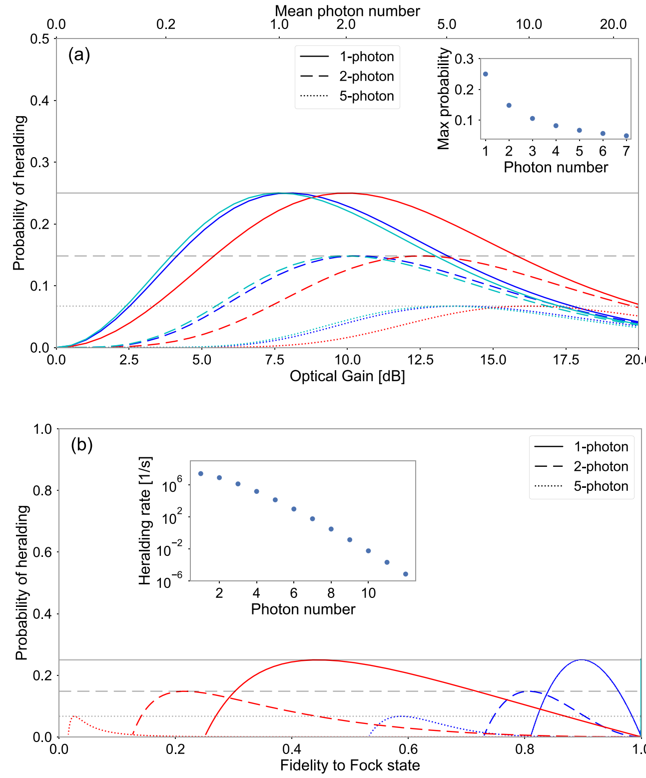

FIG. 2. Heralding probability and state fidelity are calculated for the heralding transmission values of 1.0 (cyan), 0.9 (blue), and 0.5 (red) and a heralded single-photon, two-photon, and five-photon state. (a) Dependence of heralding probability on the initial optical gain. The inset shows the maximal generation probability of an n-photon state for a single-mode source. This value is also indicated by gray horizontal lines in both figures. (b) Relation of heralding probability and state fidelity while the optical gain is changed. For a heralding transmission value of 1.0 (cyan) unit fidelity is always reached. The bottom inset shows the generation rate for Fock states under optimistic experimental assumptions (see the main text).

$$p_{\max,K=1}(n) = \frac{n^n}{(1+n)^{1+n}}. \tag{5}$$

Besides this fundamental limit, as a second step we have to consider losses, which are unavoidable for any practical system. We will consider the fidelity to the desired Fock state in a single spectral mode as a measure for the quality of the heralded state [29]. As used in Ref. [18] we will use the heralding probability and the fidelity as the benchmark parameters.

The effect of losses in the heralding arm (idler) is shown in Fig. 2, where perfect transmission is assumed for the signal. Heralding probability and state fidelity are plotted versus the optical gain which is equivalent to the squeezing value for a single-mode state. The blue curve assumes a heralding efficiency of 90%, whereas the red curve is calculated for a heralding efficiency of 50%. It can be seen that losses in the idler arm are not critical for the heralding probability because the photon number distribution of the heralding mode (heralded mode is traced out) is a thermal state and thermal states stay thermal under losses. In principle, a lower transmission value can always be counteracted by increasing the squeezing parameter, i.e., increasing the pump power of the SPDC process. The fidelity, on the other hand, is obviously influenced by losses as photon number correlations between the signal and idler mode are affected. The effect of loss in the heralding arm can be decreased by reducing the squeezing value; unit fidelity can always be reached in principle for vanishing generation probability. Figure 2(b) illustrates the fundamental limits of SPDC for higher-order Fock state generation as well as the trade-off between generation probability and state fidelity (the heralding probability is limited and cannot reach the maximum simultaneously with the fidelity). This effect becomes more pronounced for higher photon numbers and for higher losses in the heralding arm. In order to calculate the highest Fock state n that can be generated realistically with SPDC, we assume optimistic values for an experiment. We choose a fidelity of 90% with respect to the desired Fock state as a threshold for acceptable quality. We do not consider losses in the signal arm and assume perfect emission into a single Schmidt mode. For the heralding efficiency we assume an efficiency of $\eta = 90\%$. This is above the highest value that has been shown in the literature (see, e.g., Ref. [30]), but may be achievable in principle. In order to acquire enough statistics in a reasonable time we aim for a heralding rate of 0.1 event/s. The maximal repetition rate of the experiment is limited by the laser system as well as the detection system. Typical laser systems used for these experiments are Ti:sapphire lasers, which offer repetition rates around 10^8 pulses/s, as well as high energies in transform-limited pulses. We chose this value as an optimistic value for the repetition rate although the detection part of the experiment may be slower. The inset of Fig. 2(b) shows the exponentially decaying heralding rate versus the photon number under these optimistic assumptions. As an example it can be seen that only Fock states up to $n = 9$ can be realized at 0.1 event/s.

As a third step, we need to investigate the effects from multiple spectral modes. In general, the heralding will project a multispectral two-mode squeezed state into a state of definite photon number n occupying an incoherent superposition of spectral modes, which we refer to as a mixed multimode photon number state (MMPNS). In contrast, the desired state is a pure photon number state in a single mode (Fock state), which requires (inherently lossy) spectral filtering [31] or spectrally engineering the initial two-mode squeezed state to reduce the number of modes. Calculating the fidelity for the heralded multimode state is a nontrivial task which is explained in detail in the Supplemental Material [25]. The impact of multimodeness is illustrated in Fig. 3. Here, the generation probability and state fidelity are shown versus the optical gain, which is used as the generalization of the squeezing parameter for the multimode case [cf. Eq. (4)]. The maximal generation probability increases with more modes present as the mode distribution goes from being a thermal state to a Poissonian distribution $p_{\max}(n) = e^{-n}n^n/n!$. However, this increase comes at the cost of generating a more mixed photon number state and in fact decreases substantially the fidelity to the desired Fock state. In contrast to loss, spectral multimodeness cannot be counteracted by lower optical gain, which means that the Schmidt number limits the fidelity, as shown in the inset of Fig. 3. The additional decay in the

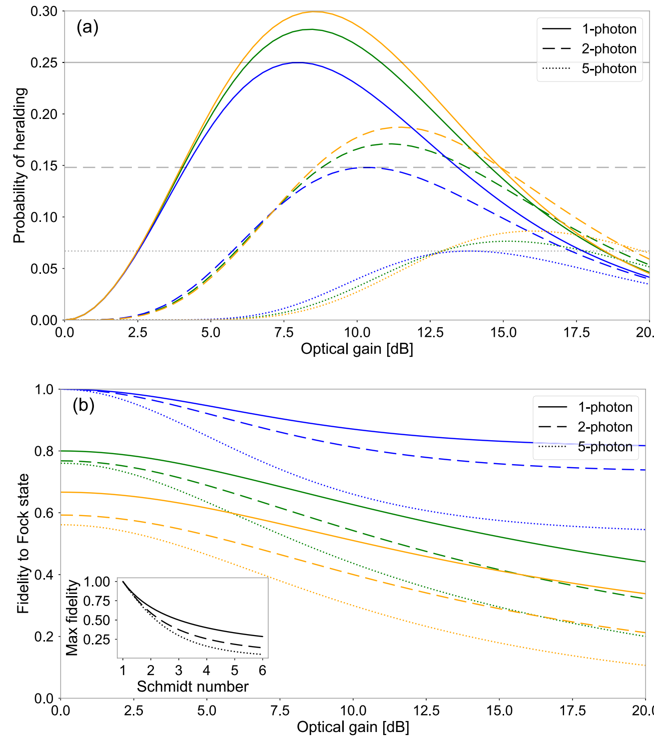

FIG. 3. (a) Heralding probability and (b) state fidelity vs the optical gain parameter. For these plots the heralding transmission value is kept constant at 0.9. Three different Schmidt values of 1, 1.5, and 2 are plotted (blue, green, and yellow curves, respectively). The inset (b) shows how the Schmidt number influences the maximal fidelity. Spectral modes up to $K_{\max} = 35$ were considered for these calculations.

fidelity for increasing optical gain (Fig. 3) can be explained by higher photon number contributions in case of heralding loss.

III. EXPERIMENTAL SETUP AND RESULTS

To confirm our theoretical findings, we experimentally generated higher-order Fock states and measured their heralding probabilities and fidelities. For this we used a type-II parametric down-conversion process (PDC) in a periodically poled potassium titanyl phosphate (KTP) waveguide. This crystal is pumped with pulsed light from a Ti:sapphire oscillator at 767.5 nm (see Fig. 4). The source is used for its highly single-mode performance [20] and extremely high brightness [16]. The spectral purity of this source depends on the phase matching and pump parameters [32]. Here, we use a nonoptimal pump bandwidth in order to see an increased effect of multiple spectral modes [33]. Detection is performed with intrinsic photon number resolving transition edge sensors [34] (TESs). These detectors offer near unit efficiency and extremely high photon number discrimination in the few-photon regime [35]. Details about the conversion from TES response functions to photon numbers can be found in the Supplemental Material [25].

We analyzed the heralding probability and state fidelity for four different pump intensities. The results are shown in Fig. 5. Heralding probabilities are analyzed for up to seven

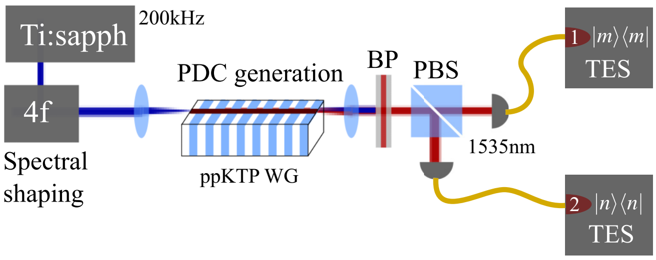

FIG. 4. Experimental setup as used in Ref. [16]. Pulsed light from a Ti:sapphire laser is spectrally filtered by a $4f$ line and coupled into a periodically poled KTP waveguide. A bandpass (BP) filter is used to filter out the pump as well as to suppress sinc sidelobes of the phase-matching function. The signal and idler are split on a polarizing beam splitter (PBS) coupled into fibers and detected by intrinsic photon number resolving transition edge sensors (TESs).

photons. State fidelities are evaluated for up to four photons on the heralding detector. Beyond four photons, the statistical uncertainties as well as errors arising from photon number identification (cf. Ref. [35]) become substantial. Colored lines show fitted curves based on the theory shown above. Only one set of fitting parameters was used for all curves (Schmidt number $K = 1.61$ and $\eta_i = 0.59$ and $\eta_s = 0.64$ for the transmission of the idler and signal arm, respectively). The experimental data can be described with very high precision with our theory. Error bars are discussed in the Supplemental Material [25]. Small deviations can be most likely explained by coupling drifts in the setup between the four measurement runs.

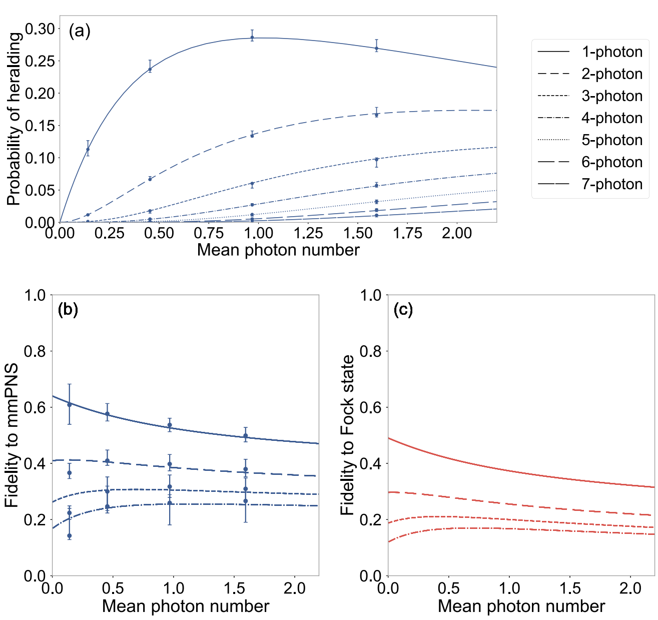

FIG. 5. Experimental data (points) for (a) the heralding probability and (b) the state fidelity without correcting for losses vs the measured mean photon number. Four different pump intensities were investigated. Colored lines show the theoretical values with $K = 1.61$, $\eta_i = 0.59$, and $\eta_s = 0.64$ as the only free parameters. In (b) the state fidelity to a mixed multimode photon number state is plotted whereas the calculated state fidelity to a Fock state is shown in (c).

The setup shown in Fig. 4 is not able to distinguish spectral modes to determine the fidelity to a Fock state directly. Instead, the convolution of all modes was measured and compared to theory with $K = 1.61$ (compare Fig. 5). The corollary of this is that this measurement can be used to extract the modal behavior even without a mode resolving measurement, by fitting the experimental data with our multimode theoretical model. In contrast to Ref. [36], this approach is valid for the strong pump regime where the mean photon number is larger than one (a similar approach has also been shown in Ref. [37]).

It can be seen that the measured heralding probability for different heralded Fock states is higher than the single-mode theory predicts [e.g., the one-photon curve in Fig. 5(a) goes above 25%]. This shows the importance of considering a multimode theory even though fewer than two Schmidt modes are present. The theory in this Rapid Communication excludes time ordering, but when searching for time ordering phenomena the effects presented here should be considered. Additionally, it can be seen that rather high mean photon numbers are required to herald higher-order Fock states with reasonable probability. In order to generate these squeezing values in a single-pass configuration, waveguided nonlinear materials are essential. If losses for the signal (heralded photon) are included (as shown in Fig. 5), more optical gain can be beneficial to increase the state fidelity. In this case losses and higher-order contributions increase the generation probability of the desired state. However, for low signal losses, which are required for most applications where higher-order Fock states are involved, the state fidelity is strictly decreasing for higher optical gain.

IV. CONCLUSION

Spontaneous parametric down-conversion is the most widely used tool to generate higher-order optical Fock states. While previous work has mostly elaborated on the structure of the multimode SPDC state and its impact on single-photon generation, the preparation of higher-order Fock states brings additional, nontrivial challenges. In this Rapid Communication the limitations of this process in terms of generation probability and state fidelity under the consideration of losses and spectral multimodeness were investigated. We identified fundamental limits for the generation of Fock states in SPDC as well as practical restrictions arising from experimental imperfections. These cause a trade-off between generation probability and state fidelity for low signal loss. This means that high-fidelity Fock states under realistic experimental constraints and generation probabilities cannot be realized beyond $n = 9$ at 0.1 event/s. We have experimentally investigated different pump intensities to maximize the generation probability of higher-order Fock states and showed that waveguided spectrally engineered sources offer many essential advantages for pulsed SPDC as they allow for high generation probabilities and single-mode emission. With this result it is possible to calculate the feasibility of experiments requiring higher-order Fock states. At the same time this stresses that alternative approaches to generate higher-order Fock states, as, for example, shown in Refs. [9,10], need to be explored further.

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[1] A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri, and P. Grangier, Generation of optical "Schrödinger cats" from photon number states, Nature (London) **448**, 784 (2007).

[2] M. J. Holland and K. Burnett, Interferometric Detection of Optical Phase Shifts at the Heisenberg Limit, Phys. Rev. Lett. **71**, 1355 (1993).

[3] T. Nagata, R. Okamoto, J. L. O'Brien, K. Sasaki, and S. Takeuchi, Beating the standard quantum limit with four-entangled photons, Science **316**, 726 (2007).

[4] S. Slussarenko, M. M. Weston, H. M. Chrzanowski, L. K. Shalm, V. B. Verma, S. W. Nam, and G. J. Pryde, Unconditional violation of the shot-noise limit in photonic quantum metrology, Nat. Photonics **11**, 700 (2017).

[5] Y. Yamamoto and H. A. Haus, Preparation, measurement and information capacity of optical quantum states, Rev. Mod. Phys. **58**, 1001 (1986).

[6] G. M. D'Ariano, L. Maccone, M. G. Paris, and M. F. Sacchi, Optical Fock-state synthesizer, Phys. Rev. A **61**, 053817 (2000).

[7] K. Sanaka, Linear optical extraction of photon-number Fock states from coherent states, Phys. Rev. A **71**, 021801 (2005).

[8] K. R. Brown, K. M. Dani, D. M. Stamper-Kurn, and K. B. Whaley, Deterministic optical Fock-state generation, Phys. Rev. A **67**, 043818 (2003).

[9] K. T. McCusker and P. G. Kwiat, Efficient Optical Quantum State Engineering, Phys. Rev. Lett. **103**, 163602 (2009).

[10] B. L. Glebov, J. Fan, and A. Migdall, Photon number squeezing in repeated parametric downconversion with ancillary photon-number measurements, Opt. Express **22**, 20358 (2014).

[11] A. Ourjoumtsev, R. Tualle-Brouri, and P. Grangier, Quantum Homodyne Tomography of a Two-Photon Fock State, Phys. Rev. Lett. **96**, 213601 (2006).

[12] E. Waks, E. Diamanti, and Y. Yamamoto, Generation of photon number states, New J. Phys. **8**, 4 (2006).

[13] A. Zavatta, V. Parigi, and M. Bellini, Toward quantum frequency combs: Boosting the generation of highly nonclassical light states by cavity-enhanced parametric down-conversion at high repetition rates, Phys. Rev. A **78**, 033809 (2008).

[14] M. Cooper, C. Söller, and B. J. Smith, High-stability time-domain balanced homodyne detector for ultrafast optical pulse applications, J. Mod. Opt. **60**, 611 (2013).

[15] T. S. Iskhakov, V. C. Usenko, U. L. Andersen, R. Filip, M. V. Chekhova, and G. Leuchs, Heralded source of bright multi-mode mesoscopic sub-Poissonian light, Opt. Lett. **41**, 2149 (2016).

[16] G. Harder, T. J. Bartley, A. E. Lita, S. W. Nam, T. Gerrits, and C. Silberhorn, Single-Mode Parametric-Down-Conversion States with 50 Photons as a Source for Mesoscopic Quantum Optics, Phys. Rev. Lett. **116**, 143601 (2016).

[17] A. M. Brańczyk, T. C. Ralph, W. Helwig, and C. Silberhorn, Optimized generation of heralded Fock states using parametric down-conversion, New J. Phys. **12**, 063001 (2010).

[18] A. Christ and C. Silberhorn, Limits on the deterministic creation of pure single-photon states using parametric down-conversion, Phys. Rev. A **85**, 023829 (2012).

[19] J. N. Quesada Mejia, Very nonlinear quantum optics, Ph.D. thesis, University of Toronto, 2015.

[20] A. Eckstein, A. Christ, P. J. Mosley, and C. Silberhorn, Highly Efficient Single-Pass Source of Pulsed Single-Mode Twin Beams of Light, Phys. Rev. Lett. **106**, 013603 (2011).

[21] A. Christ, B. Brecht, W. Mauerer, and C. Silberhorn, Theory of quantum frequency conversion and type-II parametric down-conversion in the high-gain regime, New J. Phys. **15**, 053038 (2013).

[22] N. Quesada and J. E. Sipe, Time-Ordering Effects in the Generation of Entangled Photons Using Nonlinear Optical Processes, Phys. Rev. Lett. **114**, 093903 (2015).

[23] W. Mauerer, M. Avenhaus, W. Helwig, and C. Silberhorn, How colors influence numbers: Photon statistics of parametric down-conversion, Phys. Rev. A **80**, 053815 (2009).

[24] M. F. Neuts, *Matrix-Geometric Solutions in Stochastic Models: An Algorithmic Approach* (Courier Corporation, North Chelmsford, MA, 1981).

[25] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.100.041802 for details about the Schmidt decomposition, error bars, thermal states under losses, and fidelity calculations.

[26] A. M. Brańczyk, A. Fedrizzi, T. M. Stace, T. C. Ralph, and A. G. White, Engineered optical nonlinearity for quantum light sources, Opt. Express **19**, 55 (2011).

[27] F. Graffitti, P. Barrow, M. Proietti, D. Kundys, and A. Fedrizzi, Independent high-purity photons created in domain-engineered crystals, Optica **5**, 514 (2017).

[28] A. B. U'Ren, K. Banaszek, and I. A. Walmsley, Photon engineering for quantum information processing, Quantum Inf. Comput. **3**, 480 (2003).

[29] R. Jozsa, Fidelity for mixed quantum states, J. Mod. Opt. **41**, 2315 (1994).

[30] L. K. Shalm, E. Meyer-Scott, B. G. Christensen, P. Bierhorst, M. A. Wayne, M. J. Stevens, T. Gerrits, S. Glancy, D. R. Hamel, M. S. Allman, K. J. Coakley, S. D. Dyer, C. Hodge, A. E. Lita, V. B. Verma, C. Lambrocco, E. Tortorici, A. L. Migdall, Y. Zhang, D. R. Kumor, W. H. Farr, F. Marsili, M. D. Shaw, J. A. Stern, C. Abellán, W. Amaya, V. Pruneri, T. Jennewein, M. W. Mitchell, P. G. Kwiat, J. C. Bienfang, R. P. Mirin, E. Knill, and S. W. Nam, Strong Loophole-Free Test of Local Realism, Phys. Rev. Lett. **115**, 250402 (2015).

[31] E. Meyer-Scott, N. Montaut, J. Tiedau, L. Sansoni, H. Herrmann, T. J. Bartley, and C. Silberhorn, Limits on the heralding efficiencies and spectral purities of spectrally filtered

single photons from photon-pair sources, Phys. Rev. A **95**, 061803(R) (2017).

[32] W. P. Grice, A. B. U'Ren, and I. A. Walmsley, Eliminating frequency and space-time correlations in multiphoton states, Phys. Rev. A **64**, 063815 (2001).

[33] M. Avenhaus, H. B. Coldenstrodt-Ronge, K. Laiho, W. Mauerer, I. A. Walmsley, and C. Silberhorn, Photon Number Statistics of Multimode Parametric Down-Conversion, Phys. Rev. Lett. **101**, 053601 (2008).

[34] A. E. Lita, A. J. Miller, and S. W. Nam, Counting near-infrared single-photons with 95% efficiency, Opt. Express **16**, 3032 (2008).

[35] P. C. Humphreys, B. J. Metcalf, T. Gerrits, T. Hiemstra, A. E. Lita, J. Nunn, S. W. Nam, A. Datta, W. S. Kolthammer, and I. A. Walmsley, Tomography of photon-number resolving continuous-output detectors, New J. Phys. **17**, 103044 (2015).

[36] A. Christ, K. Laiho, A. Eckstein, K. N. Cassemiro, and C. Silberhorn, Probing multimode squeezing with correlation functions, New J. Phys. **13**, 033027 (2011).

[37] I. A. Burenkov, A. K. Sharma, T. Gerrits, G. Harder, T. J. Bartley, C. Silberhorn, E. A. Goldschmidt, and S. V. Polyakov, Full statistical mode reconstruction of a light field via a photon-number-resolved measurement, Phys. Rev. A **95**, 053806 (2017).